MAT 4510
MANDATORY ASSIGNMENT

Deadline: *You must turn in your paper before Thursday, October 24., 2013, 2.30 p.m. at "Ekspedisjonskontoret", Matematisk institutt (7th floor NHA), or in the specially designated box in the 7th floor. Remember to use the official front page available at*

http://www.mn.uio.no/math/studier/admin/obligatorisk-innlevering/obligforside-eng.pdf

If you due to illness or other circumstances want to extend the dead line, you must apply for an extension to you must apply for an extension to Robin Bjørnetun Jacobsen (room B718, NHA, e-mail:studieinfo@math.uio.no, phone 22 85 58 82). Remember that illness has to be documented by a medical doctor! See

http://www.mn.uio.no/math/studier/admin/obligatorisk-innlevering/obligregelverk-eng.html

for more information about the rules for mandatory assignments

Instructions: *The assignment is compulsory, and students who do not get both their paper accepted, will not get access to the final exam. To get this assignment accepted, you need a score of at least 50 %. In the evaluation, credit will be given for a clear and well-organized presentation. All questions (problem 1a) , 3b) etc.) have equal weight. Students who do not get their original paper accepted, but who have made serious and documented attempts to solve the problems, will get one chance of turning in an improved version.*

Problem 1

a) Find the hyperbolic distance

$$d_{\mathbb{H}}(-1+\sqrt{3}i, 1+\sqrt{3}i).$$

b) Find $h \in \text{Möb}^+(\mathbb{H})$ such that $h(i) = 1+\sqrt{3}i$ and $h(3i) = -1+\sqrt{3}i$.

Problem 2

a) Let

$$f(z) = \frac{3z+1}{-2z}.$$

Classify f (i.e. decide whether f is parabolic, hyperbolic or elliptic) and write f explicitly as a conjugate of a map on normal form.

b) Let

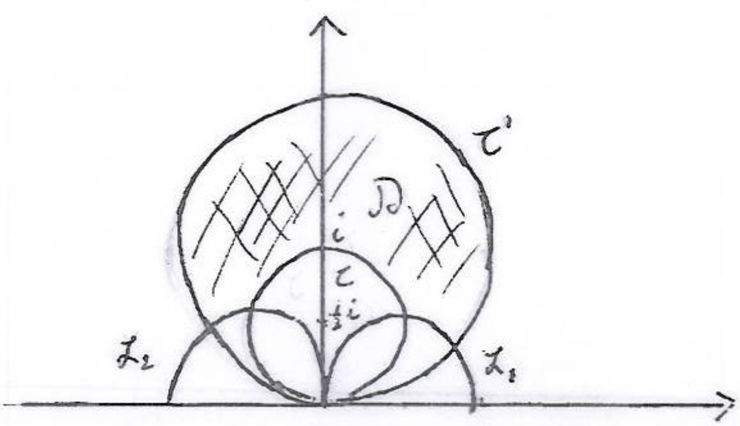
$$f(z) = \frac{\bar{z}+2}{2\bar{z}+1}.$$

Find the fix points of f. Find an inversion g and a map $h \in \text{Möb}^+(\mathbb{H})$ such that $gh = hg = f$.

Problem 3

a) Find $h \in \text{Möb}^+(\mathbb{H})$ such that $h(\mathcal{C}) = \mathcal{L}$
where $\mathcal{C} = \{z : |z - \frac{i}{2}| = \frac{1}{2}\}$, and $\mathcal{L} = \{t + i : t \in \mathbb{R}\} \cup \{\infty\}$.

b) Let $\mathcal{D}$ be the domain in $\mathbb{H}$ bounded by the circle $\mathcal{C}$ (given in a)), the circle $\mathcal{C}' = \{z : |z - i| = 1\}$, and the two $\mathbb{H}$-lines $\mathcal{L}_1 = \{z : |z - \frac{1}{2}| = \frac{1}{2}\}$ and $\mathcal{L}_2 = \{z : |z + \frac{1}{2}| = \frac{1}{2}\}$ (se the figure given below). Calculate the hyperbolic area of $\mathcal{D}$ (hint: Use h from a)).



Problem 4

a) Consider a hyperbolic triangle with angles α, β and γ and opposite sides of hyperbolic length a, b and c respectively . Assume that $b = c = \rho$ and $\alpha = \frac{\pi}{2}$. Find expressions for β and γ in terms of ρ.

b) Consider the domain $\mathcal{U}$ in the first quadrant of $\mathbb{D}$, bounded by the circle $\{z : |z| = r\}$ and the $\mathbb{D}$-line $\overleftrightarrow{r\,ri}$ (see the figure below). Let r be such that the hyperbolic triangle with vertices $0, r$ and ri have angles $\frac{\pi}{2}, \frac{\pi}{6}$ and $\frac{\pi}{6}$ respectively. Calculate the hyperbolic area of $\mathcal{U}$.