

MAT4510

Mandatory assignment 1 of 1

Submission deadline

Thursday 20th October 2022, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with $\text{\LaTeX}$). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) no later than the same day as the deadline.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

In order to pass the assignment you need a score of at least 66%.

Problem 1. Let $c : \mathbb{R} \rightarrow \mathbb{R}^2$ be the curve defined by

$$c(t) = (1 + 2 \cos t) \cdot (\cos t, \sin t).$$

(i) Show that c has period 2π .

(ii) Compute the winding number n_c (which was called the *rotation index* in the lectures).

Hint: Express c in terms of the complex exponential function

$$e^{it} = \cos t + i \sin t.$$

Problem 2. Let

$$U = \{(u, v, t) \in \mathbb{R}^3 \mid u^2 + v^2 < 1\}$$

and define the function $f : U \rightarrow \mathbb{R}$ by

$$f(u, v, t) = u \sin t - v \cos t.$$

(i) Show that $\Sigma = f^{-1}(0)$ is a regular surface (i.e. a 2-dimensional submanifold of $\mathbb{R}^3$).

(ii) Show that the map $F : \mathbb{R} \times (-1, 1) \rightarrow \Sigma$ given by

$$F(\theta, s) = (s \cos \theta, s \sin \theta, \theta)$$

is a diffeomorphism.

(iii) Compute the Gauss curvature of Σ .

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Problem 3. Let U, Σ be as in Problem 1 and define the map $\phi : U \rightarrow \mathbb{R}^3$ by

$$\phi(u, v, t) = ((1 + u) \cos 2t, (1 + u) \sin 2t, v).$$

Note that the image $\phi(U)$ is obtained by rotating the disk $(x - 1)^2 + z^2 < 1$ in the xz -plane about the z -axis.

(i) Show that $S = \phi(\Sigma)$ is a regular surface.

(ii) Prove that S is non-orientable.

Problem 4. Let X, Y be the vector fields on the sphere

$$S^2 = \{(u, v, w) \in \mathbb{R}^3 \mid u^2 + v^2 + w^2 = 1\}$$

given by

$$\begin{aligned} X_{(u,v,w)} &= (-v, u, 0), \\ Y_{(u,v,w)} &= (0, -w, v). \end{aligned}$$

Compute the covariant derivative $\nabla_X Y$.