

MAT4510

Mandatory assignment 1 of 1

Submission deadline

Thursday 6th November 2025, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with L^AT_EX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you must contact the lecturer (e-mail: kfroyshov@math.uio.no) no later than the same day as the deadline.

All assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

Problem 1 Let M and N be manifolds with $M \subset N$, and let $R : N \rightarrow M$ be a smooth retraction. Show that for all $p \in M$ and $v \in T_p M$ one has $dR_p(v) = v$.

Problem 2 Let S be a regular surface with Gauss curvature K and Riemannian curvature tensor R . Suppose $K = 0$. Show that $R = 0$.

Problem 3 Let S be a regular surface and $c : I \rightarrow S$ a smooth curve of unit speed, where I is an interval. Suppose N is a smooth vector field on S along c such that for all $t \in I$ one has

$$\|N(t)\| = 1, \quad \langle \dot{c}(t), N(t) \rangle = 0.$$

Prove the following.

(a) There is a smooth function $\kappa : I \rightarrow \mathbb{R}$ such that

$$\frac{\nabla \dot{c}}{dt} = \kappa N, \quad \frac{\nabla N}{dt} = -\kappa \dot{c}.$$

(b) Let $\phi : I \rightarrow \mathbb{R}$ be a primitive of $-\kappa$, i.e. $\dot{\phi} = -\kappa$. Then

$$W(t) := \cos \phi(t) \cdot \dot{c}(t) + \sin \phi(t) \cdot N(t)$$

is a parallel vector field on S along c .

Problem 4 Let

$$S := \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = z^2 + 1\}.$$

(a) Show that S is a surface of revolution.

(b) Calculate the Gauss curvature of S .

(c) Calculate the parallel transport along the curve

$$c : [0, 2\pi] \rightarrow S, \quad c(\theta) = (\sqrt{2} \cos \theta, \sqrt{2} \sin \theta, 1)$$

using the results of Problem 3.