

## Manifolds, V2018

Problem sheet 1, to be discussed on Monday the 22nd January 2018.

**Problem 1.** Tu, Problem 6.1.

**Problem 2.** Show that  $\mathbb{R}^n \setminus \{0\}$  is diffeomorphic to  $S^{n-1} \times \mathbb{R}$  for every positive integer  $n$ .

**Problem 3.** Let  $m, n$  be integers such that  $m \geq 0, n > 0$ , and let  $V \subset \mathbb{R}^{m+n}$  be an  $m$ -dimensional linear subspace. Show that  $\mathbb{R}^{m+n} \setminus V$  is diffeomorphic to  $S^{n-1} \times \mathbb{R}^{m+1}$ .

**Problem 4.** Let  $A := \mathbb{R} \times \{0, 1\}$ , where  $\mathbb{R}$  has the usual topology and  $\{0, 1\}$  has the discrete topology. Let  $L$  be the quotient of  $A$  obtained by identifying  $(x, 0)$  with  $(x, 1)$  for  $x \neq 0$ .

- (i) Show that every point in  $L$  has a neighbourhood homeomorphic to an open subset of  $\mathbb{R}$ .
- (ii) Show that  $L$  is not Hausdorff.