

Manifolds, V2018

Problem sheet 8, to be discussed on Monday the 19th March 2018.

Problem 1. Let G be a smooth manifold equipped with a group structure such that the multiplication map $\mu : G \times G \rightarrow G$ is smooth.

(i) Show that the tangent map

$$\mu_* : T_{(e,e)}(G \times G) \rightarrow T_e G$$

is given by $\mu_*(A, B) = A + B$, where e is the identity element of G and we identify $T_{(e,e)}(G \times G) = T_e G \oplus T_e G$.

(ii) Show that the inversion map $\iota : G \rightarrow G$ is also smooth.

Hint: First prove that the map

$$G \times G \rightarrow G \times G, \quad (g, h) \mapsto (g, gh)$$

is a diffeomorphism.

(iii) Show that the tangent map

$$\iota_* : T_e G \rightarrow T_e G$$

is given by $\iota_*(A) = -A$.

Problem 2. Find all left-invariant vector fields on the Lie group \mathbb{R}^n .