

MAT4520

Mandatory assignment 1 of 1

Submission deadline

Thursday 23rd April 2026, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with $\text{\LaTeX}$). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

In order to pass the assignment you need to show a serious attempt at solving at least half of the 8 (sub)problems.

Problem 1. For $n \geq 2$ let $\pi : S^{n-1} \rightarrow \mathbb{RP}^{n-1}$ be the projection and let A be a symmetric $n \times n$ real matrix.

(i) Show that there is a unique smooth function $f : \mathbb{RP}^{n-1} \rightarrow \mathbb{R}$ such that for any $x \in S^{n-1}$ one has

$$f(\pi(x)) = \langle Ax, x \rangle,$$

where $\langle \cdot, \cdot \rangle$ is the usual scalar product on $\mathbb{R}^n$.

(ii) Suppose A has distinct eigenvalues. Show that f has exactly n critical points.

Problem 2. Let m, n, r be positive integers. Let $M := M(m \times n, \mathbb{R})$ be the space of $m \times n$ real matrices and S the subset of M consisting of matrices of rank r . Show that S is a submanifold of M and compute its dimension.

Problem 3. Compute the flow of the following vector field on $\mathbb{R}^2$:

$$V = y \frac{\partial}{\partial x} + \frac{\partial}{\partial y}.$$

Problem 4. Let $G \subset M(3 \times 3, \mathbb{R})$ be the space of all matrices of the form

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

where $a, b, c \in \mathbb{R}$.

- (i) Show that G is a regular submanifold of $M(3 \times 3, \mathbb{R})$ diffeomorphic to $\mathbb{R}^3$.
- (ii) Show that G is a group under matrix multiplication, and that this makes G a Lie group.
- (iii) Find a basis v_1, v_2, v_3 for the tangent space $T_I G$, where I is the identity matrix.
- (iv) For $i < j$ express $[v_i, v_j]$ as a linear combination of v_1, v_2, v_3 . (Here, $T_I G$ has the Lie algebra structure induced from the Lie algebra of G .)