

MAT4530 — Algebraic Topology I

Mandatory assignment 1 of 1

Submission deadline

Thursday April 21, 2022, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with $\text{\LaTeX}$). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) no later than the same day as the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

A score of about 40% is required to pass.

Exercise 1. If $n > 1$, show that any map $\mathbb{R}P^n \rightarrow S^1$ is nullhomotopic.

Exercise 2. Let $GL_n(\mathbb{R})$ denote the general linear group, consisting of invertible $n \times n$ -matrices with real coefficients. Show that $GL_n(\mathbb{R})$ deformation retracts onto the subgroup $O(n)$ of orthogonal matrices.

Exercise 3 (Pushouts and homotopy).

- (a) Write S^1 as the pushout of $* \leftarrow \partial I \rightarrow I$. Applying π_1 , show that the resulting square is not a pushout square in the category of groups. Thus π_1 does not preserve pushouts.
- (b) Find an example to show that π_1 does not preserve pullbacks either.
- (c) Although the unit disc D^n is homotopy equivalent to a point, show that the pushouts of the diagrams

$$\begin{array}{ccc} S^{n-1} & \hookrightarrow & D^n \\ \downarrow & & \\ * & & \end{array} \qquad \begin{array}{ccc} S^{n-1} & \longrightarrow & * \\ \downarrow & & \\ * & & \end{array}$$

are not homotopy equivalent.

- (d) Define the *homotopy pushout* of

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ g \downarrow & & \\ Z & & \end{array}$$

as

$$Y \amalg (X \times I) \amalg Z / \sim,$$

where

$$(x, 0) \sim f(x), \quad (x, 1) \sim g(x) \quad \text{for all } x \in X.$$

- (e) Compute the homotopy pushouts of $X \leftarrow X \times Y \rightarrow Y$ and $* \leftarrow X \rightarrow *$, where the maps in the first diagram are the projections.
- (f) Show that the homotopy pushouts of the diagrams in (c) are homotopy equivalent.

Exercise 4. Exercise 1.2.9 in Hatcher.

Exercise 5. Exercise 2.1.6 in Hatcher.

Exercise 6. Exercise 2.1.9 in Hatcher.