

# MAT4530 (2023 SPRING) CHECKLIST

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## Framework for spaces

- Explain formal definitions of
  - cell complexes (CW complexes)
  - $\Delta$ -complexes
- Explain basic ideas behind these frameworks.
- How are they different?
- Give realization of
  - spheres
  - surfaces
  - projective spaces
- What is a pointed space?
- What is a pair of spaces / cell complexes?

## Operations

- In the setting of topological spaces, explain the formal definitions of
  - wedge sum
  - smash product
  - suspension
- Give examples for these operations.
- Describe these operations in the framework of cell /  $\Delta$ -complexes.

## Intermediate concepts

How do you model the following concepts?

- paths in spaces
- loops in spaces
- simplexes in spaces

## Homotopy

- Explain the formal definitions of
  - homotopy equivalence of maps
  - homotopy equivalence of paths
  - homotopy equivalence of spaces
- Give concrete examples of above concepts.

## Invariants

- Explain the ideas behind the following invariants:
  - fundamental group
  - singular homology
  - relative (singular) homology
  - simplicial homology
  - cellular homology
  - local homology
- How are they related?
- How are they different? When do you see the difference?
- How do you see homotopy invariance for each of these?

## Computations

- Explain the ideas behind:
  - Van Kampen's theorem
  - homology long exact sequence for pairs
  - degree of maps  $S^n \rightarrow S^n$
  - excision for singular homology
  - Mayer–Vietoris exact sequence
- Use these to compute the above invariants for:
  - bouquet spaces  $S^1 \vee \cdots \vee S^1$
  - spheres
  - surfaces
  - real projective spaces