MAT4530 (2023 SPRING) MANDATORY ASSIGNMENT

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Typeset your solutions to these problems by LaTeX and submit the file on the course `canvas` page. Due date is March 31.

Problem 1 (Section 1.1, Exercise 7)**.** Let $X = S^1 \times I$. Consider the map $X \to X$ given by

$$f(\cos\theta, \sin\theta, s) = (\cos(\theta + 2\pi s), \sin(\theta + 2\pi s), s).$$

(1) Show that f is homotopic to the identity map of X by a homotopy $F\colon X \times I \to X$ such that

$$F(\cos\theta, \sin\theta, 0, t) = (\cos\theta, \sin\theta, 0) \quad (t \in I). \tag{1}$$

(2) Similarly, show that f is homotopic to the identity map of X by a homotopy $F\colon X \times I \to X$ such that

$$F(\cos\theta, \sin\theta, 1, t) = (\cos\theta, \sin\theta, 1) \quad (t \in I). \tag{2}$$

(3) Show that there is no homotopy $F\colon X \times I \to X$ between f and the identity map of X satisfying both (1) and (2). (One way to think about this: Look at the result of gluing the two copies of S^1 in X sitting as $S^1 \times \{0\}$ and $S^1 \times \{1\}$. Then you can look at the path $g\colon I \to X$ given by $g(t) = f(\cos\theta_0, \sin\theta_0, t)$ for a fixed θ_0.)

Problem 2 (Section 1.2, Exercise 9)**.** Here M_g is the closed oriented surface of genus g, and M'_g is the complement of an open disk in M_g (punctured surface). Let C be a cycle in M_g that separates M_g to punctured surfaces M'_h and M'_k, with $\partial M'_h = C = \partial M'_k$.

(1) Show that M'_h does not retract onto C.
(2) Show that M_g retracts onto the 'meridian' cycle C' along any hole. (Picture proof is enough.)

Problem 3 (Section 1.3, Exercises 1 and 2)**.** Here $p\colon \tilde{X} \to X$ denotes a covering.

(1) Given $p\colon \tilde{X} \to X$ and a subspace $A \subset X$, show that $\tilde{A} = p^{-1}(A)$ becomes a covering over A.
(2) Given $p_1\colon \tilde{X}_1 \to X_1$ and $p_2\colon \tilde{X}_2 \to X_2$, show that the direct product space $\tilde{X}_1 \times \tilde{X}_2$ becomes a covering over $X_1 \times X_2$.
(3) Use these to show that, given $p_1\colon \tilde{X}_1 \to X$ and $p_2\colon \tilde{X}_2 \to X$, the fiber product space $\tilde{X}_1 \times_X \tilde{X}_2$ becomes a covering over X.

Problem 4 (Section 2.1, Exercise 3)**.** (1) Construct a Δ-complex structure on S^n whose vertices are the points $(0, \dots, \pm 1, \dots, 0) \in S^n \subset \mathbb{R}^{n+1}$.
(2) Construct a Δ-complex structure on $\mathbb{R}P^n$ as a quotient of this structure.

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