

MAT4530

Mandatory assignment

Submission deadline

Thursday 10th April 2025, 14:30 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with L^AT_EX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) no later than the same day as the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

Each of the five problems carry equal weight. A score over 50 % is sufficient to pass. You are welcome to cooperate with other students, but your answers should reflect your own understanding.

Problem 1. Let $\mathcal{C}$ be a category, and $a, b \in \mathcal{C}$ any objects. Let $f : a \rightarrow b$ be any morphism. Prove that a pushout $a \amalg_a b$ of the diagram $a \xleftarrow{\text{id}} a \xrightarrow{f} b$ exists and is isomorphic to b .

Problem 2. The additivity axiom for homology can be phrased as saying that H_k takes coproducts in Top to coproducts in Ab . Consider the following analogous claim for pushouts:

If

$$\begin{array}{ccc} A & \longrightarrow & B \\ \downarrow & & \downarrow \\ C & \longrightarrow & X \end{array}$$

is a pushout diagram in Top , then

$$\begin{array}{ccc} H_k(A) & \longrightarrow & H_k(B) \\ \downarrow & & \downarrow \\ H_k(C) & \longrightarrow & H_k(X) \end{array}$$

is a pushout diagram in Ab .

Prove or disprove this claim.

Problem 3. Given a commutative diagram of abelian groups

$$\begin{array}{ccccccc} A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & D \\ \downarrow f & & \downarrow g & & \downarrow h & & \downarrow i \\ A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & D' \end{array}$$

with exact rows, surjective f and injective i , prove the following:

- (a) If h is surjective then g is surjective.
- (b) If g is injective then h is injective.

Problem 4. Recall that we defined $\mathbb{R}P^n$ as quotient $S^n / \sim$, where $x \sim -x$, i.e. we identify opposite points.

- (a) Give $\mathbb{R}P^2$ the structure of a Δ -complex, i.e. describe a Δ -set S and a homeomorphism $|S| \cong \mathbb{R}P^2$.

- (b) Use the description from (a) to compute homology of $\mathbb{R}P^2$. (Compute homology of the Δ -set constructed in (a)).

(Of course, in (a), different choices of S are possible.)

Problem 5. Let X be a topological space, and $U_0, \dots, U_n \subseteq X$ open subsets with $\bigcup_{i=0}^n U_i = X$, and the following property: For any subset $I \subseteq \{0, \dots, n\}$, the intersection $\bigcap_{i \in I} U_i$ is a disjoint union of contractible spaces.

Prove that $H_k(X) = 0$ for $k > n$. (Hint: Mayer-Vietoris, induction)