

Solution (Hatcher Section 3.1, Exercise 1). For $f : H \rightarrow H'$ and chosen resolutions $P_* \rightarrow H \rightarrow 0$ and $P'_* \rightarrow H' \rightarrow 0$, the Lemma we proved in the lecture provides us with a map $f_* : P_* \rightarrow P'_*$ extending it. Furthermore, this is unique up to chain homotopy. Dualizing, we obtain a map $f^* : (P'_*)^* \rightarrow P^*$, still unique up to chain homotopy, and so a well-defined induced map on cohomology, $\text{Ext}^n(H, G) \rightarrow \text{Ext}^n(H', G)$. Given $f : H \rightarrow H'$ and $f' : H' \rightarrow H''$, we see $(f'' \circ f')^* = (f')^* \circ (f'')^*$, and similarly $\text{id}^* = \text{id}$. So this gives $\text{Ext}^n(-, G)$ the structure of a contravariant functor.

In the other argument, we proceed similarly, but it is easier: For $f : G \rightarrow G'$, we obtain an induced map $\text{Hom}(P_*, G) \rightarrow \text{Hom}(P_*, G')$, and an induced map on cohomology $\text{Ext}^n(H, G) \rightarrow \text{Ext}^n(H, G')$. Again compatibility with composition and identity are clear, and so $\text{Ext}^n(H, -)$ is a covariant functor.

Solution (Hatcher Section 3.1, Exercise 2). Postcomposition with the multiplication by n map $G \rightarrow G$ induces multiplication by n on $\text{Hom}(A, G)$ for any abelian group A . So for a resolution $P_* \rightarrow H \rightarrow 0$, it induces multiplication by n on $P^* = \text{Hom}(P_*, G)$. The multiplication by n map on a cochain complex induced multiplication by n on its cohomology.

Similarly, the multiplication by n map $H \rightarrow H$ may be extended to a resolution $P_* \rightarrow H \rightarrow 0$ by letting all $f_i : P_i \rightarrow P_i$ be the multiplication by n map. Dualizing, we have the multiplication by n map on each $(P_i)^*$, since precomposition with the multiplication by n map on $\text{Hom}(P_i, G)$ is simply the multiplication by n map (as $f(nx) = nf(x)$). The multiplication by n map on P^* again induces the multiplication by n map on cohomology.

Solution (Hatcher Section 3.1, Exercise 3). We may take the sequence

$$\dots \xrightarrow{2} \mathbb{Z}/4 \xrightarrow{2} \mathbb{Z}/4 \rightarrow \mathbb{Z}/2 \rightarrow 0$$

as our resolution. We have that $\text{Hom}(\mathbb{Z}/4, \mathbb{Z}/2) \cong \mathbb{Z}/2$, and so we obtain as P^* the cochain complex

$$\mathbb{Z}/2 \xrightarrow{2} \mathbb{Z}/2 \xrightarrow{2} \mathbb{Z}/2 \rightarrow \dots,$$

which has all cohomology groups given by $\mathbb{Z}/2$ (since the multiplication by 2 map is zero here!). So $\text{Ext}_{\mathbb{Z}/4}^n(\mathbb{Z}/2, \mathbb{Z}/2) \cong \mathbb{Z}/2$ for all $n \geq 0$.