Mandatory assignment MAT4640 -Spring 2012

Deadline 08.03.2012 , 14.30

Your solution may be

- mailed electronically to the lecturer with the "subject" Mand4640-Your Name
- delivered by hand to the lecturer
- put in the reception box for mandatory assignments at the department.

In all cases, the paper should be marked with your name and MAT4640.
The lecturer's e-mail address is: dnormann@math.uio.no.

The deadline is Thursday 24.03.2011.
The set consists of three problems. In order to pass, you must have made serious attempts on all three problems, and successful attempts on at least 50% of the set.
You may cooperate, but your final paper shall be written by yourself, and reflect your understanding at the time of delivery, see the general rules of the department.

Problem 1

Let f and g be maps from ω to ω.
We say that $f \leq g$ a.e. (almost everywhere) if

$$\{n \in \omega \mid g(n) < f(n)\}$$

is finite.

a) Use Martin's axiom to show that if $X \subseteq {}^{\omega}\omega$ has cardinality $< |2^{\omega}|$ then there is an $f : \omega \to \omega$ such that $g \leq f$ a.e. for all $g \in X$.

b) Use a) to show that 2^{ω} is a regular cardinal under the assumption of Martin's axiom.

Problem 2

In this problem we work with **ZFC**.

a) Show that $R(\omega+\omega)$ satisfies all axioms of **ZFC** except the replacement scheme.

b) Find an instance of the replacement scheme that does not hold in $R(\omega + \omega)$.
Justify your choice of an example.
If you use defined symbols in your example, explain briefly why these symbols can be interpreted in $R(\omega + \omega)$.

Problem 3

Let κ be an infinite cardinal and let X be the set of all functions from κ to ω.
We let $f \prec g$ if $f(\alpha) < g(\alpha)$ for the least $\alpha < \kappa$ such that $f(\alpha) \neq g(\alpha)$.

a) Show that $\prec$ is a total ordering, but not a well ordering.

b) Show that if R is a total ordering on κ, then there is a function

$$f : \kappa \to X$$

such that for all $\alpha < \kappa$ and $\beta < \kappa$ we have

$$\alpha R \beta \Leftrightarrow F(\alpha) \prec F(\beta).$$

c) Show that κ^+ is the supremum of the order types of the well ordered subsets of X. Always let $\prec$ induce the ordering on the subsets.

d) Discuss if we can conclude from this that $\kappa^+ \leq \kappa^\omega (= |{}^\omega\kappa|)$ for all infinite cardinals κ.

Dag Normann