

MAT4735 - Advanced Financial Modelling

ASSIGNMENT

Evaluation: passed/not passed

To receive a passing grade all points in the exercises have to be presented and at least 50% correct material. In the evaluation, weight is also given to the presentation.

Delivery of the assignment

You can choose between scanning handwritten notes or typing the solution directly on a computer with a typesetting software for mathematics. It is expected that you give a clear presentation with all necessary explanations. The assignment must be submitted as a single PDF file. It is your responsibility to guarantee a good readable pdf file. Please remember to number the pages.

All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If you work in collaboration, please write the names of the team on the front page (this secures faster correction).

Assignment: About subordination and stochastic volatility.

This exercise is all about a subclass of Lévy processes, which is known as subordinators. These are often used as stochastic time-change, in simulation techniques, and they also find application in stochastic volatility modelling. See, e.g., the Barndorff-Nielsen and Shephard (BNS) model. The main reference for this assignment is the book by Cont and Tankov, see below. You can also find other material within the course literature.

1. Give a definition of subordinator.
2. Give the characteristic function (Lévy-Khintchine formula) and the characteristic triplet of a subordinator.
3. Using the points above, show that the Poisson process is a subordinator.
4. Prove the following statement: *Let $X = X_t$, $t \geq 0$, be a Lévy process. Define*

$$S_t := \sum_{s \leq t; \Delta X_s \neq 0} (\Delta X_s)^2, \quad t \geq 0.$$

The process above is a subordinator. See [CT, Proposition 3.11].

5. Let $B = B_t$, $t \geq 0$, be a Brownian motion and $S = S_t$, $t \geq 0$, a subordinator independent of B . Consider the process

$$X_t := B_{S_t}, \quad t \geq 0.$$

This is called *subordinated Brownian motion* or *time-changed Brownian motion*. This process is a Lévy process. Describe its characteristic function and characteristic triplet following [CT, Theorem 4.2].

6. The BNS stochastic volatility model is based on a linear stochastic differential equation driven by a subordinator Z :

$$dV_t = -\lambda V_t dt + Z_t; \quad V_0 > 0 \quad (\lambda > 0).$$

Using the Itô formula and the Lévy-Itô representation, verify that the solution of this equation is given by

$$V_t = e^{-\lambda t} V_0 + \int_0^t e^{\lambda(s-t)} dZ_s.$$

You can read about this model in [CT, Section 15.3 and onward].

References

- [CT] Rama Cont and Peter Tankov: *Financial Modelling with Jump Processes*, Chapman and Hall 2004.