Exercise sheet 1

MAT4760, Spring 2016

Lectures 1, 2 and 3

Exercise 1

Compute the mean and variance of a Poisson process

Exercise 2

Compute the mean and variance of a compound Poisson process, where you state conditions for the existence of these two.

Exercise 3

A probability distribution F on the real line is said to be *infinitely divisible* if for every natural number $n \geq 1$ there exists n IID random variables $\{X_{ni}\}_{i=1}^n$ such that their sum $X_{n1} + \ldots + X_{nn}$ has distribution F.

a) Show that the standard normal distirbution is infinitely divisible

b) Show that the distribution of $L(t)$, where L is a Levy process, must be infinitely divisible for any $t > 0$.

Exercise 4

Develop an algorithm that simulates paths of a compound Poisson process.

Exercise 5

Compute the cumulant of a compound Poisson process with exponentially distributed jumps.

Exercise 6

Let L be a pure jump Levy process with jumps smaller than or equal to $\epsilon < 1$. Compute its variance.

Exercise 7

Subordination of Brownian motion: Let $B(t)$ be a Brownian motion and $U(t)$ a so-called subordinator, meaning a Levy process with increasing paths. Show that $L(t) = B(U(t))$ is a Levy process, and compute its cumulant function. Show that if $U(1)$ is inverse Gaussian distributed, then L becomes a normal inverse Gaussian Levy process.