

1 Linear filtering in discrete time

- Linear projection of one r.v. onto another one.
- Statement of Theorem 1.1 (not the proof) (Orthogonal projection)
- Statement and proof (case that the covariance matrix is non-singular) of Proposition 1.3.
- Recursive orthogonal projection: statement and proof (only sketch) of Proposition 1.4 .
- Setup and statement of the Kalman-Bucy filter.

2 Nonlinear filtering in discrete time

- Review of conditional expectation: know the main properties of conditional expectation.
- Statement and proof of Bayes formula and its version when the conditioning σ -algebra is generated by a random variable. Theorem 2.17 and Corollary 2.18.
- Setup, statement and proof of the filtering recursion, Theorem 2.22.
- Change of measure method setup and proof.
- The Kalman-Bucy when the noises and initial conditions are Gaussian. Statement of Lemma 2.26. Proof of only point (3) in that lemma.

3 Review on stochastic analysis

In the exam the basic results will not be asked explicitly, but you need to know them for solving the problems. You need to know how to use Itô's formula, Girsanov's theorem, martingale representation theorem, integration by parts (or product formula) and Lévy's characterization theorem.

4 Linear filtering in continuous time

- One dimensional case: statement of the setup and main result Theorem 4.1 (No proof)
- General case: statement of the setup and main result Theorem 4.4.
- In this part, besides the previous two points, you can be asked to solve exercises such as the ones proposed for this lecture.

5 Nonlinear filtering in continuous time

- Statement of the main setup: Including the signal as a solution of a martingale problem, specification of the observation process, statement of the problem Definition 5.2 and main general result Theorem 5.4
- Innovation processes: Definition 5.7 and Proposition 5.8 with proof.
- FKK martingale representation. Theorem 5.10 with proof.
- The change of probability measure method. Explain the change of measure, Lemma 5.15 (no proof), reasoning and sketch of proof of Proposition 5.16.
- Kallianpur-Striebel formula: Proposition 5.19 statement and proof. This includes Proposition 5.17 (no proof), the definition of the unnormalized distribution, Lemma 5.21 and Corollary 5.23.
- The Zakai equation statement and sketch of proof. Definition 5.20, Lemmas 5.21 and 5.25, and Corollary 5.23 without proofs.
- The Kushner-Stratonovich equation statement and sketch of proof.
- Forget about the part regarding the innovation approach.

6 Particle methods

- This topic is not part of the exam.

7 Particle filter in discrete time

All the references below are respect to: Bain and Crisan. (2009) Fundamentals of Stochastic Filtering. Springer Verlag

- Setup and description of the problem (pages 257-259 BC09) and statement of main result Proposition 10.6 (no proof), describing the steps of prediction and updating.
- Convergence of approximations to π_t : The fixed observation case (section 10.3.1) Theorem 10.7 with proof
- Convergence of approximations to π_t : The random observation case (section 10.3.2) Proposition 10.14 with proof (note that this one builds on the proof of Theorem 10.7)

- Setup and description of the algorithm in section 10.4 + properties of the multinomial distribution as the offspring distribution in the algorithm.
- Convergence of the algorithm. Theorem 10.27+Corollary 10.28 both with proof.