7th October, 2026

COURSE MAT4800, Fall 2026

Mandatory assignment 1 of 1

Submission deadline

Thursday 8th October 2026, 11:00 in Canvas (canvas.uio.no).

Instructions

Note that you have **one attempt** to pass the assignment. This means that there are no second attempts.

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) no later than the same day as the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

The score to pass is 55%**.**

In some of the problems, the following numbers are needed:

α_0 = the last digit of your student ID

β_0 = the second-to-last digit of your student ID

Define $\alpha = \alpha_0 + 1$, $\beta = \beta_0 + 2$.

For example, if your student ID is 1234567890, then $\alpha_0 = 0$ and $\beta_0 = 9$. then $\alpha = \alpha_0 + 1 = 1$ and $\beta = \beta_0 + 2 = 11$.

Look on canvas.uio.no for the precise student ID. If you do not know, then you can ask me.

What are the last two digits of your student ID, α_0, β_0, α and β?

Last two digits of student ID =

$\alpha_0 =$

$\beta_0 =$

$\alpha =$

$\beta =$

Problem 1. Recall that $\mathbf{P}^2$ is covered by 3 open sets: $U_1 = \{[X : Y : Z] \in \mathbf{C}^2 : \ Z \neq 0\}$, $U_2 = \{[X : Y : Z] \in \mathbf{C}^2 : \ Y \neq 0\}$, and $U_3 = \{[X : Y : Z] \in \mathbf{C}^2 : \ X \neq 0\}$.

Let $E_{aff} \subset \mathbf{C}^2$ be the curve defined by the equation: $E_{aff} = \{(x, y) \in \mathbf{C}^2 : \ y^4 = x^3 + x^2 + \alpha xy + 1\}$.

Let $\pi : E_{aff} \to \mathbf{C}$ be defined by $\pi(x, y) = x^2 + \alpha x + \beta y + 1$.

1) (10 points) What is the homogenization of the equation defined E_{aff}? Let $E \subset \mathbf{P}^2$ be defined by that equation.

2) (10 points) Is E smooth? If it is not smooth, then what are its singular points?

3) (10 points) Let E_{sm} be the smooth part of E. Show that π can be extended as a holomorphic map π_1 from E_{sm} to $\mathbf{P}^1$. What is the formula for π_1 on $E_{sm} \cap U_2$?

4) (10 points) At which point of E_{sm} that apparently π does not have a definite value. Find the value of π at such points.

5) (10 points) What is the maximal and minimal cardinality of the sets $\pi_1^{-1}(x)$, $x \in \mathbf{P}^1$?

6) (10 points) Does π_1 have any branch points? List all such points?

7) (10 points) Which points on E at which π apparently is not defined? List them and provide the reason why, and work out whether it can be defined at such points.

Problem 2. Notations are as in Problem 1. Let V_1 and V_2 be the two standard open sets in $\mathbf{P}^1$, defined by: $V_1 = \{[X : Y] : Y \neq 0\}$ and $V_2 = \{[X : Y] : X \neq 0\}$.

1) (10 points) Let ω_1 be the holomorphic differential 1-form on V_1, defined by $\omega_1 = (x/(1 - x))dx$, where $x = X/Y$. Show that ω_1 can be extended to a meromorphic differential 1-form on $\mathbf{P}^1$. Call ω this extension.

2) (10 points) What are zeros and poles of ω? Compute the multiplicity of ω at these points?

3) (10 points) Compute the residues of ω at its poles.

4) (10 points) Let $\hat{\omega} = \pi_1^*(\omega)$. Compute $\hat{\omega}$.

5) (10 points) What are zeros and poles of $\hat{\omega}$? Compute the multiplicity of $\hat{\omega}$ at these points?

6) (10 points) Compute the residues of $\hat{\omega}$ at its poles.

Problem 3. Consider $f(x,T) = T^2 - xT + x^{\beta+1}$, for $x \in \mathbf{C}$. Call $F(x,T)$ the extension of this to $\mathbf{P}^1$.

1) (10 points) What is the formula of $F(x,T)$ in the open set $V_2 \subset \mathbf{P}^1$?

2) (10 points) Describe the Riemann surface $\mathcal{X}$ which is the analytic extension of $F(x,T)$.

3) (10 points) Explain why $\mathcal{X}$ is compact.

4) (10 points) Compute the Euler characteristic of $\mathcal{X}$.

5) (10 points) Compute the genus of $\mathcal{X}$.