20th February, 2017

COURSE MAT4760/9760

Mandatory assignment 1 of 1

Submission deadline

Thursday 2nd March 2017, 14:30 in the mandatory activity hand-in box, situated on the 7th floor of Niels Henrik Abels hus.

Instructions

You may write your answers either by hand or on a computer (for instance with LaTeX). All submissions must include the following official front page:

http://www.uio.no/english/studies/admin/compulsory-activities/mn-math-obligforside-eng.pdf

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you understand the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code. In order to print the code from one of the Linux machines belonging to the university, move to the folder containing your program and type

```
lpr -P pullprint_manufacturer filename
```

where `filename` is the file you wish to print and `pullprint_manufacturer` is the name of the manufacturer of the printer you wish print from. Common choices are `pullprint_Ricoh` and `pullprint_HP`.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (7th floor of Niels Henrik Abels hus, e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

The assignment is mandatory to take part to the final examination. In order to pass, all question must be answered satisfactory.

Problem 1. Let $X \sim \mathcal{N}(\mu, \sigma^2)$ defined on some probability space $(\Omega, \mathcal{F}, P)$.

1. Compute $\psi(\theta) = \mathbb{E}[e^{\theta X}]$ for $\theta \in \mathbb{R}$.
2. Define $L(X;\theta) := e^{\theta X}/\psi(\theta)$ and show that $Q_\theta(A) = \mathbb{E}[L(X;\theta)\mathbf{1}_A], A \in \mathcal{F}$ defines a probability measure on $(\Omega, \mathcal{F})$. Show that $Q_\theta \ll P$.
3. Find the law of X under Q_θ, that is, the law of X as a random variable defined on $(\Omega, \mathcal{F}, Q_\theta)$.
4. Let $\mathcal{G}$ be a sub-σ-algebra of $\mathcal{F}$. Show that for any $Y \in L^1(\Omega, \mathcal{F}, Q_\theta)$ one has that

$$\mathbb{E}_{Q_\theta}[Y|\mathcal{G}] = \frac{\mathbb{E}[YL(X;\theta)|\mathcal{G}]}{\mathbb{E}[L(X;\theta)|\mathcal{G}]}, \quad Q_\theta\text{-a.s.}$$

Problem 2. Let X_0, V, W be independent standard normal random variables and $a, b \in \mathbb{R}$. Consider

$$X = aX_0 + V,$$
$$Y = bX + W.$$

Find

$$E\left[\varphi(X)\,|\sigma(Y)\right],$$

for any bounded, real function φ.

Problem 3. Let X be an $\mathbb{F}$-adapted Markov process with values in a finite state space I. The $Q = \{q_{ij}(t), i, j \in I, t \geq 0\}$ matrix is defined so that for all $t, h \geq 0$

$$P\left(X_{t+h} = j | X_t = i\right) = J_i(j) + q_{ij}(t)\,h + o(h),$$

as h tends to zero, uniformly in t, for any $i, j \in I$, where J_i is the indicator function of the atom i. Assume that Q has teh following properties

1. $q_{ii}(t) \leq 0$ for all $i \in I$, $q_{ij}(t) \geq 0$ for all $i, j \in I, i \neq j$.
2. $\sum_j q_{ij}(t) = 0$ for all $i \in I$.
3. $\sup_{t \geq 0} |q_{ij}(t)| < \infty$ for all $i, j \in I$.