UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in	MEK 1100 — Vector calculus.
Day of examination:	Wednesday 12th of June 2013.
Examination hours:	09:00 – 13:00.
This problem set consists of 3 pages.	
Appendices:	Two pages with formulas.
Permitted aids:	K. Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 10 individual questions. Each individual question is given a score from 0 to 10 points (10 for complete answer, 0 for blank). Maximum possible score is 100 points. Check that you do not forget any of the questions.

Problem 1

A vector field $\boldsymbol{v}$ is defined by the potential $\phi = x^2 - y^2$. Find the vector field $\boldsymbol{v}$. Find the stream function ψ if it exists. Show that the field has a stagnation point (where $\boldsymbol{v} = 0$). Find the stream lines through the stagnation point. Make a sketch where the stagnation point is marked by $\bullet$, stream lines are marked by solid lines, equipotential curves are marked by dashed lines, and the direction of the field is marked by arrows.

Problem 2

A fluid with density ρ flows in the xy-plane in the velocity field of a point vortex given in respectively polar and Cartesian coordinates by

$$\boldsymbol{v} = \frac{A}{r}\boldsymbol{i}_\theta = \frac{A}{x^2+y^2}(-y\boldsymbol{i} + x\boldsymbol{j})$$

where $x = r\cos\theta$ and $y = r\sin\theta$.

2a

Compute the circulation around a circle in the xy-plane with radius R and center in the origin.

(Continued on page 2.)

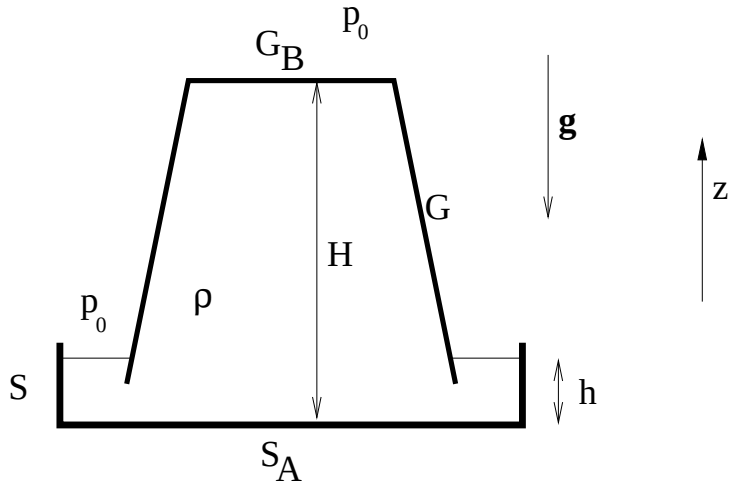
2b

Compute the acceleration of a fluid particle in this field.

2c

Compute the pressure. We suppose that the pressure force is the only force that acts and that the pressure far from the origin is p_0.

Problem 3



A glass (G) is filled with water and is held with the bottom G_B up and with the opening down toward a bowl (S) without coming into contact with the bowl. The bottom of the bowl (S_A) is horizontal with area A. The bottom of the glass (G_B) is horizontal with med area B. The water has height h from the bottom of the bowl to free air, and has height H from the bottom of the bowl to the bottom of the glass. The air has constant pressure p_0 everywhere. The water has density ρ. The acceleration of gravity $\boldsymbol{g}$ is oriented downwards. The z-axis points upwards.
Find the pressure in the water just below the bottom of the glass G_B. Find the pressure force that acts from the water on the bottom of the glass G_B.

Problem 4

In a rectangular room the temperature at some time is given by

$$T = T_0 + A(xa - x^2)(yb - y^2)z \qquad \text{for} \quad 0 \le x \le a, \quad 0 \le y \le b, \quad 0 \le z \le c$$

where T_0 and A are constants. Here the x- and y-axes are horizontal and the z-axis points upwards. We suppose that the air in the room is at rest, has heat conductivity k and heat diffusivity κ.

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The heat flux density is given by Fourier's law $\boldsymbol{H} = -k\nabla T$, and the heat equation is given by

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T.$$

4a

Give the physical units (dimensions) of A, k, κ, T and $\boldsymbol{H}$.

4b

A mosquito hovers in the middle of the room $\left\{x = \frac{a}{2}, y = \frac{b}{2}, z = \frac{c}{2}\right\}$. In which direction must the mosquito fly in order to move to colder temperatures as fast as possible?

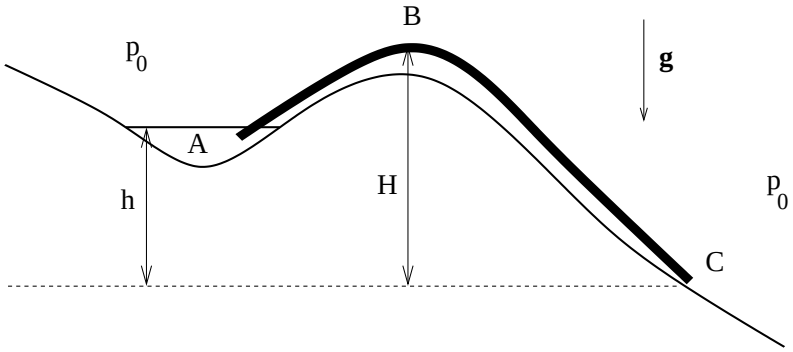
4c

Find out if the temperature distribution given above is an equilibrium solution of the heat equation. Find out if the temperature in the middle of the room $\left\{x = \frac{a}{2}, y = \frac{b}{2}, z = \frac{c}{2}\right\}$ will stay constant, increase or decrease.

4d

Show that the total heat flux out through all walls, floor and ceiling can be written as a volume integral over the entire room. Compute the heat flux.

Problem 5



In order to provide water, a pipe has been extended from a lake (A) with water surface at a height h above the location (C) where the water is tapped. The pipe is laid over a hill (B) and it has been necessary to pump the water through the pipe in order that it runs as in a siphon. The highest point of the pipe (B) is at a height H above the tapping location (C). Determine the flow speed and the volume flux when we suppose stationary frictionless flow and that the pipe has constant cross section S.

END