UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in:	MEK 1100 — Vector calculus.
Day of examination:	Thursday 1 June 2017.
Examination hours:	14:30 – 18:30.
This problem set consists of 3 pages.	
Appendices:	Two pages with formulas.
Permitted aids:	K. Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 10 individual questions. Each individual question is given a score from 0 to 10 points (10 for complete answer, 0 for blank). Maximum possible score is 100 points. Check that you do not forget any of the questions.

Problem 1

A vector field is given in dimensionless form by

$$\boldsymbol{v} = 4x^2y\boldsymbol{i} + xyz\boldsymbol{j} + yz^2\boldsymbol{k}$$

1a

Compute the integrated flux out of the closed surface enclosing the box $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $0 \leq z \leq 1$. Do this as a surface integral.

1b

Show that you get the same answer by computing a volume integral by means of a suitable integral theorem.
What is the name of the integral theorem?

Problem 2

The water in a river with rectangular cross-section $-a \leq x \leq a$ and $-h \leq z \leq 0$ flows in the y-direction with the velocity field

$$\boldsymbol{v} = \frac{U(z+h)}{h}\boldsymbol{j}$$

(Continued on page 2.)

where U is the speed of the water at the surface, h is the depth, $2a$ is the width, $\boldsymbol{j}$ is the unit coordinate vector in the y-direction, the z-axis points vertically up and the xy-plane is horizontal. Where the water is in contact with air, at $z = 0$, the pressure is equal to the constant air pressure $p = p_0$. The water has constant density ρ. We do not consider friction. The acceleration of gravity $\boldsymbol{g} = -g\boldsymbol{k}$ points down.

2a

Find out if this is a potential flow and in that case find the potential.

2b

Compute the integrated volume flux through a cross-section of the river given by $y = 0$.

2c

Show that the continuity equation is satisfied.

2d

Use Euler's equation of motion to find the pressure p everywhere in the water.

2e

Compute the circulation around a closed curve given by the following rectangle: From the origin $(0, 0, 0)$ to the bottom $(0, 0, -h)$ along the bottom to $(0, b, -h)$ up to the surface $(0, b, 0)$ and back to the origin. Here b is a constant. Do this by direct computation as a curve integral.
Check that you get the same answer by computing a surface integral according to a suitable integral theorem.
What is the name of the integral theorem?

Problem 3

A carousel at an amusement park rotates with constant angular velocity $\boldsymbol{\omega} = \omega\boldsymbol{k}$. The axis of rotation coincides with the z-axis which points up, $\boldsymbol{k}$ is the unit coordinate vector in the z-direction, the xy-plane is horizontal, r is the radial horizontal distance from the axis of rotation, θ is the angle in the xy-plane such that $x = r\cos\theta$ and $y = r\sin\theta$.
A person on the carousel moves toward the center with constant speed v_0 in the radial direction, at the same time as she rotates with the movement of the carousel in the angular direction, she therefor moves according to the

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velocity field

$$\boldsymbol{v} = \omega r \boldsymbol{i}_\theta - v_0 \boldsymbol{i}_r$$

where $\boldsymbol{i}_r$ is the unit coordinate vector in the radial direction and $\boldsymbol{i}_\theta$ is the unit coordinate vector in the angular direction.

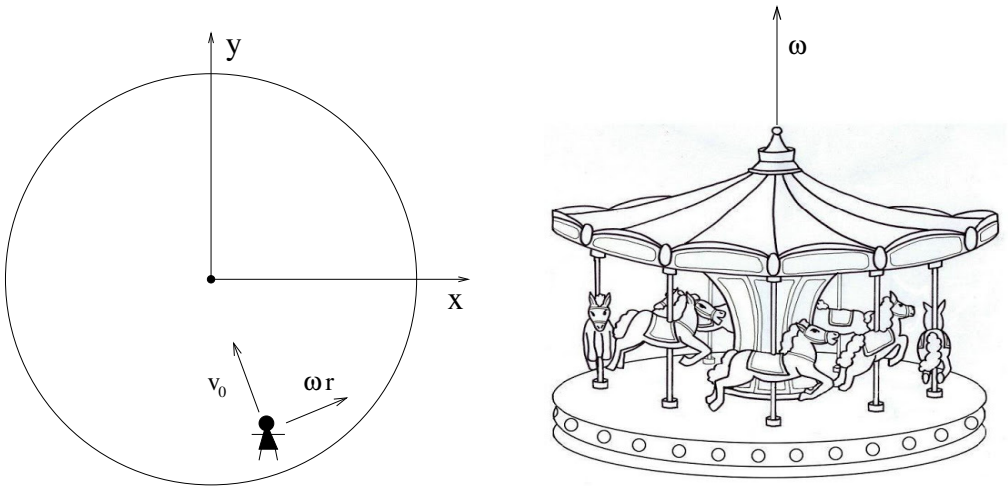


Illustration from `http://www.imagui.com/a/dibujo-de-feria-iRRjA8xLx`

Note: If you try this on a carousel that rotates freely you will experience that the angular velocity increases. In this problem, however, we assume that the carousel does not rotate freely, but that the angular velocity is kept constant!

In the following sub-problems we want the answers to be presented in cylindrical coordinates:

3a

Compute the divergence of $\boldsymbol{v}$.

3b

Compute the curl of $\boldsymbol{v}$.

3c

We shall find the acceleration of the person by computing the particle derivative of the velocity field $\boldsymbol{v}$: Compute the local acceleration and the convective acceleration.

END