UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in	MEK 1100 — Vector calculus.
Day of examination:	Thursday 14 October 2010.
Examination hours:	15:00 – 17:00.
This problem set consists of 2 pages.	
Appendices:	Formula sheet (2 pages).
Permitted aids:	K. Rottmann: Matematische Formelsamlung, certified calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are a total of 8 partial problems. Each partial problem is given a score from 0 to 12.5 (12.5 for complete answer, 0 for blank). The maximum possible score is 100. Check that you do not forget any of the questions.

Problem 1

The function $h(x, y)$ is given by

$$h(x, y) = e^{-x^2-y^2}.$$

The function h describes the height above the sea of a terrain $z = h(x, y)$, where x and y are horizontal coordinates and z is vertical coordinate.

1a

Find the gradient of h.

1b

Find the directional derivative of h at the point $(x = 1, y = 1)$ for the direction toward the origin $(x = 0, y = 0)$.

1c

Find the unit normal vector of the terrain for arbitrary values of x and y. Let the normal vector point in positive z-direction.

(Continued on page 2.)

Problem 2

A vector field is given by

$$\boldsymbol{v} = x\boldsymbol{i}$$

where $\boldsymbol{i}$ is unit coordinate vector in the x-direction.

2a

Find the divergence and curl of $\boldsymbol{v}$.

2b

Find by direct computation the circulation of $\boldsymbol{v}$ around a circle with unit radius centered at the origin.

2c

Does the field $\boldsymbol{v}$ have a stream function? Explain your answer.

2d

Find the stream lines of $\boldsymbol{v}$ (show how thay are computed). Sketch a couple of stream lines in the xy-plane, and on each of these denote with arrows $(>, <)$ the direction of the flow and with filled circles $(\bullet)$ any possible stagnation points (where $\boldsymbol{v} = 0$).

2e

The vector field $\boldsymbol{v}$ flows through a box with corners (0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0) and (1,1,1). Here all the corners are given by their (x, y, z)-coordinates. Find by direct computation the flux of $\boldsymbol{v}$ out of the box.

— The end —