UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in	MEK 1100 — Vector calculus.
Day of examination:	Thursday 13 October 2011.
Examination hours:	15:00 – 17:00.
This problem set consists of 2 pages.	
Appendices:	Two pages with formulas.
Permitted aids:	K. Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 10 individual questions. Each individual question is given a score from 0 to 10 points (10 for complete answer, 0 for blank). Maximum possible score is 100 points. Check that you do not forget any of the questions.

Problem 1

We shall look at the vector field $\boldsymbol{v} = \boldsymbol{i} + x\boldsymbol{j}$.

1a

Find the curl of $\boldsymbol{v}$.

1b

Find the divergence of $\boldsymbol{v}$.

1c

Find by direct calculation the circulation of $\boldsymbol{v}$ around the circle γ which is in the xz-plane with radius 1 and is centered at the origin. (Note that it is xz and not xy!)

1d

Find out if $\boldsymbol{v}$ has a scalar potential ϕ, and in that case find the potential.

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1e

Find out if $\boldsymbol{v}$ has a stream function ψ, and in that case find the stream function.

1f

Find the stream line of $\boldsymbol{v}$ that passes through the point with position vector $\boldsymbol{r}_0 = 2\boldsymbol{i} + \boldsymbol{j}$. Sketch the stream line.

Problem 2

We shall look at the function $h(x, y) = y^2 + x$.

2a

Find the gradient of h.

2b

Find the directional derivative of h in the direction of the vector $\boldsymbol{G} = \boldsymbol{i} + \boldsymbol{j}$ at an arbitrary point (x, y).

Problem 3

We shall look at a curved surface S given by $z = y^2 - 2$. We shall let the surface S be constrained by $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Here the z-axis points vertically up, and the x- and y-axes are horizontal.
The surface S constitutes the bottom of a lake. The water in the lake is at rest and has hydrostatic pressure $p = p_0 - \rho g z$ where p_0 is a constant reference pressure, ρ is the density of the water and g is the acceleration of gravity.

3a

Find a unit normal vector $\boldsymbol{n}$ to the surface S. Let $\boldsymbol{n}$ have positive z-component.

3b

Find the total force $\boldsymbol{F}$ due to the pressure, acting from the water on the surface S.

END