UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in:	MEK 1100 — Vector calculus.
Day of examination:	Thursday 10 October 2013.
Examination hours:	15:00 – 17:00.
This problem set consists of 3 pages.	
Appendices:	Two pages with formulas.
Permitted aids:	K. Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 10 individual questions. Each individual question is given a score from 0 to 10 points (10 for complete answer, 0 for blank). Maximum possible score is 100 points. Check that you do not forget any of the questions.

Problem 1

The temperature in a classroom is given by the expression

$$T(x, y, z) = T_0 + \frac{A}{B + \frac{x^2+y^2}{R^2}} + Cz$$

Here x and y are horizontal coordinates, z is vertical coordinate, and T is the temperature in the room. The temperature is measured in Kelvin (K) and all lengths are measured in meter (m).

Explain what are the physical units of the constants A, B, C, R and T_0.

We want to make the expression non-dimensional by scaling all lengths by R and all temperatures by T_0. Let all the non-dimensional variables be marked by asterisk, for example x^*. Derive the scaled and non-dimensional expression for the temperature in the room.

Problem 2

We shall look at the vector field

$$\boldsymbol{v} = x\boldsymbol{i} - (y+1)\boldsymbol{j}$$

2a

Compute the divergence of $\boldsymbol{v}$.

(Continued on page 2.)

2b

Compute the curl of $\boldsymbol{v}$.

2c

Find out if the vector field $\boldsymbol{v}$ has a scalar potential ϕ, and find it in case it exists.

2d

Find out if the vector field $\boldsymbol{v}$ has a stream function ψ, and find it in case it exists.

2e

Find the circulation of $\boldsymbol{v}$ around the circle γ, where γ is given by $x^2+y^2=4$, by direct computation.

2f

We shall check the answer from the last problem by computing a surface integral over the surface S, where S is a dome given by $z=\sqrt{4-x^2-y^2}$, by using a suitable integral theorem.

Explain which integral theorem we must use.

Compute the unit normal vector $\boldsymbol{n}$ of the surface S.

Compute the circulation of $\boldsymbol{v}$ around the circle γ indirectly by computing a surface integral over S.

2g

Find an equation for all stream lines of $\boldsymbol{v}$.

Find all stagnation points of $\boldsymbol{v}$ (those points where $\boldsymbol{v}=\boldsymbol{0}$), and find in particular all stream lines that go through the stagnation points.

2h

Make a sketch of the vector field $\boldsymbol{v}$ where stagnation points are indicated by the symbol $\bullet$, stream lines are indicated by solid curves, the direction of the field is indicated with arrows, and the magnitude of the field is indicated by the length of the arrows. Make sure to indicate particularly those stream lines that go through the stagnation points.

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Problem 3

An aquarium is filled with water. Around the aquarium we have air with pressure p_0. The water is at rest and has density ρ. The acceleration of gravity $\boldsymbol{g}$ is directed downwards.

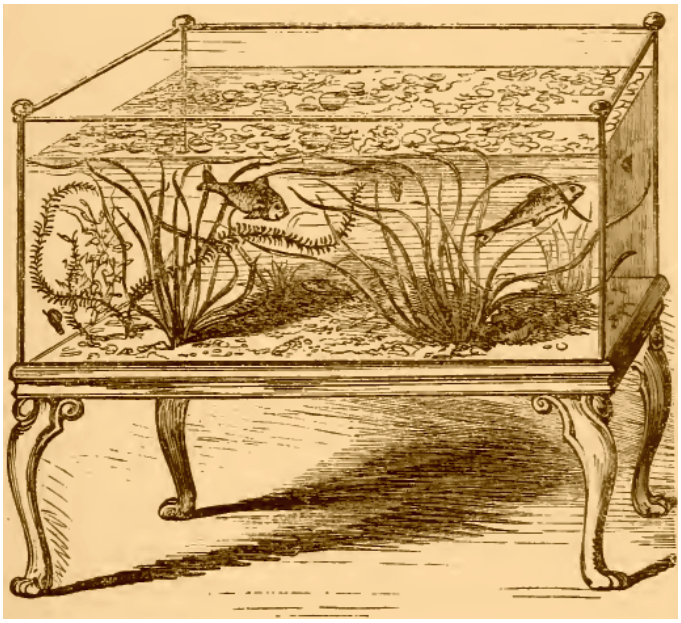
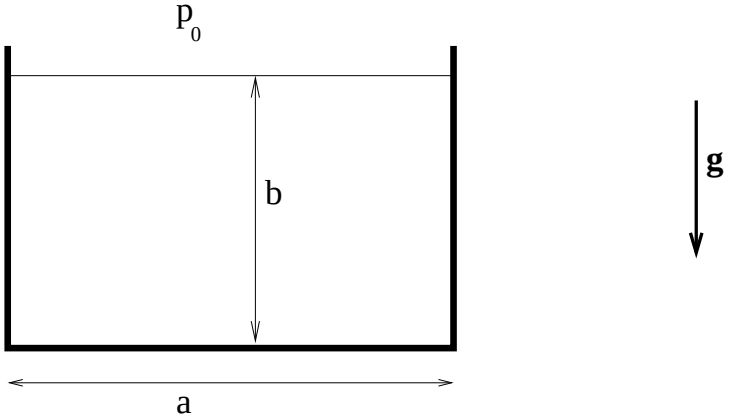


Illustration from Wikipedia:

Shirley Hibberd, The Book of the Aquarium and Water Cabinet. London: Groombridge & Sons. 1856.

We want to compute the pressure force that acts on one side of the aquarium. This side has rectangular form, with width a in horizontal direction, and the water has height b in vertical direction. We let x and y be horizontal coordinates, z be vertical coordinate, and the z-axis points up. The relevant side is shown in the figure below and in the xz-plane, the water is here in the region $y > 0$.



Compute the pressure force that acts on this side of the aquarium.

END