UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in	MEK 1100 — Vector calculus.
Day of examination:	Tuesday 29 March 2011.
Examination hours:	15:00 – 17:00.
This problem set consists of 2 pages.	
Appendices:	Two pages with formulas.
Permitted aids:	K. Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 10 individual questions. Each individual question is given a score from 0 to 10 points (10 for complete answer, 0 for blank). Maximum possible score is 100 points. Check that you do not forget any of the questions.

Problem 1

We shall look at the vector field $\boldsymbol{v} = \boldsymbol{i} - x\boldsymbol{k}$. Let the circle γ be given by $\{x^2 + y^2 = R^2,\, z = 1\}$ where the constant R is the radius of the circle.

1a

Find a suitable parametrisation for the circle γ.

1b

Find the circulation of $\boldsymbol{v}$ around the circle γ by direct computation.

1c

Compute the curl of $\boldsymbol{v}$. Has $\boldsymbol{v}$ a scalar potential ϕ? Justify the answer.

1d

Show how we can compute the circulation of $\boldsymbol{v}$ around the circle γ indirectly by means of an integral theorem.

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Problem 2

We shall look at the vector field $\boldsymbol{v} = y\boldsymbol{i} + x\boldsymbol{j} + \boldsymbol{k}$.

2a

Find the divergence of $\boldsymbol{v}$.

2b

Has the field $\boldsymbol{v}$ a stream function? Justify the answer.
Explain which scalar equation or equations one can solve in order to find the stream lines of $\boldsymbol{v}$. You should not solve the equations.

2c

Compute the flux of $\boldsymbol{v}$ through the part of the xy-plane that is bounded by $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Let positive flux be oriented in the z-directions.

Problem 3

Let the surface S be given by $z = x^2 - y^2 + 2$. We shall consider the z-axis to point up, and (x, y) to be horizontal coordinates.

3a

Find the unit normal vector $\boldsymbol{n}$ to the surface S. Let $\boldsymbol{n}$ point up.

3b

Find two tangent vectors $\boldsymbol{t}_x$ and $\boldsymbol{t}_y$ to the surface S. Let $\boldsymbol{t}_x$ and $\boldsymbol{t}_y$ have horizontal components respectively in the x-direction and y-direction only.

3c

Compute the flux of the vector field $\boldsymbol{v} = y\boldsymbol{i} + x\boldsymbol{j} + \boldsymbol{k}$ through the part of the surface S that is bounded by $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Let positive flux be oriented up.

END