UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in:	MEK 1100 — Vector calculus.
Day of examination:	Thursday 19 March 2013.
Examination hours:	15:00 – 17:00.
This problem set consists of 3 pages.	
Appendices:	Two pages with formulas.
Permitted aids:	K. Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 10 individual questions. Each individual question is given a score from 0 to 10 points (10 for complete answer, 0 for blank). Maximum possible score is 100 points. Check that you do not forget any of the questions.

Problem 1

We shall consider a swimming pool with a somewhat special shape: The bottom of the pool is given by $z = -h(x, y)$ where the depth is

$$h(x, y) = h_0 - x - \alpha y^2 \qquad \text{for} \quad x \geq 0.$$

Here x and y are horizontal coordinates, the z-axis points upward, and both α and h_0 are positive constants. At $x = 0$ there is a vertical wall. The pool is filled up with water in the region $z \leq 0$.

1a

Sketch a few depth contours (h equal to constant value) for selected depths in the interval $0 \leq h \leq h_0$. Mark in particular the depth contour $h = 0$ which corresponds to the edge of the water.
Given that h, x and y have dimension of length, explain what are the dimensions of h_0 and α.

1b

Determine in which direction the bottom of the pool has its steepest ascent at the point $x = y = 0$. Find the slope of the bottom in this direction at this point.

(Continued on page 2.)

1c

Determine the unit normal vector $\boldsymbol{n}$ of the bottom for arbitrary x and y. Let $\boldsymbol{n}$ be oriented such that it points into the water.

Problem 2

We shall consider the vector field $\boldsymbol{v} = (2x - y)\boldsymbol{i} - y^2\boldsymbol{j} - y^2 z\boldsymbol{k}$.
Let the circle γ be given by $x^2 + y^2 = 1$ and $z = 1$.

2a

Find the divergence of $\boldsymbol{v}$.

2b

Find the curl of $\boldsymbol{v}$.

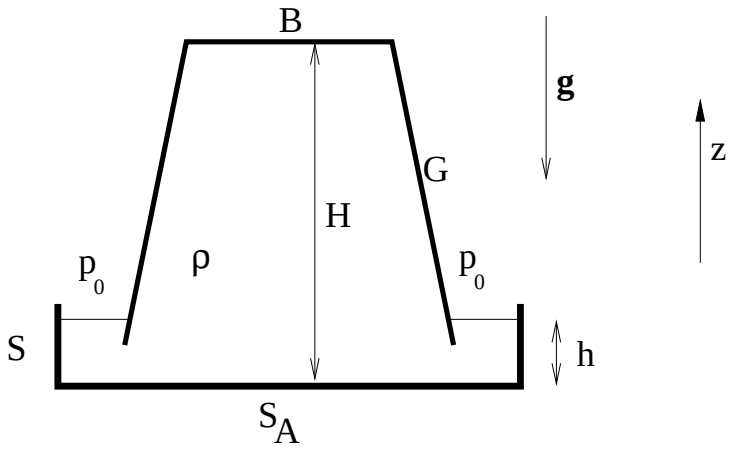
2c

Find the circulation of $\boldsymbol{v}$ around γ by direct computation.

2d

Find the circulation of $\boldsymbol{v}$ around γ indirectly by computing a surface integral over a suitable surface S. Explain which surface S you choose and which integral theorem you use.

Problem 3



A glass (G) is filled with water and is held with the bottom (B) up and with the opening down against a bowl (S) without touching the bowl. The bottom of the bowl (S_A) is horizontal with area A. The water has height h from the

(Continued on page 3.)

bottom of the bowl to free air, and has height H from the bottom of the bowl to the bottom of the glass. The air has constant pressure p_0. The water has density ρ. The acceleration of gravity $\boldsymbol{g}$ is oriented down. Let the z-axis point up.

Find the pressure in the water just below the bottom of the glass B.
Find the force that acts from the water on the bottom of the bowl S_A.

Problem 4

We shall look at the vector field

$$\boldsymbol{v} = xy\boldsymbol{i} + v_y(x, y)\boldsymbol{j}$$

where v_y is a function with the property $v_y(0, 0) = 0$.

4a

Determine $v_y(x, y)$ such that the field has no divergence and is irrotational.

4b

Find the stream function for the field and sketch the stream lines.
Mark possible stagnation points (where $\boldsymbol{v} = 0$) with the symbol $\bullet$ and mark in particular possible stream lines going through possible stagnation points.
Mark the direction of the field with arrows.

END