UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in:	MEK 1100 — Vector calculus.
Day of examination:	Tuesday 24 March 2015.
Examination hours:	11:00 – 13:00.
This problem set consists of 2 pages.	
Appendices:	Two pages with formulas.
Permitted aids:	K. Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 10 individual questions. Each individual question is given a score from 0 to 10 points (10 for complete answer, 0 for blank). Maximum possible score is 100 points. Check that you do not forget any of the questions.

Problem 1

In this entire exam set we will consider the vector field

$$\boldsymbol{v} = x^2\boldsymbol{i} - y\boldsymbol{j}$$

in the xy-plane.

1a

Compute the divergence of $\boldsymbol{v}$.

1b

Compute the curl of $\boldsymbol{v}$.

1c

Find out if the vector field $\boldsymbol{v}$ has a stream function ψ, and find the stream function in case it exists.

1d

Find out if the vector field $\boldsymbol{v}$ has a potential ϕ, and find the potential in case it exists.

(Continued on page 2.)

1e

Sketch a vector arrow plot (similar to *quiver* on the computer) for $\boldsymbol{v}$ in a neighborhood around the origin. Let the strength of the field be indicated by the length of the arrows.
Find all stagnation points (where $\boldsymbol{v} = \boldsymbol{0}$) and indicate them in the plot.

1f

Find equations that represent all streamlines of the vector field $\boldsymbol{v}$. Find in particular the streamlines that pass through the two points A $(x = 1, y = 0)$ and B $(x = 1, y = 1)$. Indicate these two streamlines in the plot of the previous problem.

1g

Compute the flux of $\boldsymbol{v}$ through the straight line segment λ between the two points A $(x = 1, y = 0)$ and B $(x = 1, y = 1)$.

1h

Find the circulation of $\boldsymbol{v}$ around the circle γ, where γ is given by $x^2 + y^2 = 1$, by direct computation.

1i

Check the answer in the previous problem by computing the circulation as a surface integral by using a suitable integral theorem. What is the name of the integral theorem you use?

1j

Let us interpret the vector field $\boldsymbol{v}$ as a velocity field, and consider a particle moving according to this velocity field. Compute the acceleration of the particle.

END