UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in:	MEK 1100 — Vector calculus.
Day of examination:	Tuesday 15 March 2016.
Examination hours:	15:00 – 17:00.
This problem set consists of 2 pages.	
Appendices:	Two pages with formulas.
Permitted aids:	K. Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 10 individual questions. Each individual question is given a score from 0 to 10 points (10 for complete answer, 0 for blank). Maximum possible score is 100 points. Check that you do not forget any of the questions.

Problem 1

The temperature in a region is given in dimensional form as

$$T = T_0 + \alpha x + \beta y^2 \tag{1}$$

where T_0 and α and β are constants. We measure T and T_0 with unit Celsius. We measure the horizontal coordinates x and y with unit meter. They are oriented such that the x-axis points east and the y-axis points north.

1a

Find the physical units for α and β.

1b

Compute the directional derivative of the temperature toward northeast at the point (x, y).

1c

Scale equation (1) such that it is brought into dimensionless form.

Problem 2

A vector field is given in dimensionless form by $\boldsymbol{v} = y\boldsymbol{i} - x\boldsymbol{j} + z\boldsymbol{k}$.

(Continued on page 2.)

2a

Compute the divergence of $\boldsymbol{v}$.

2b

Compute the curl of $\boldsymbol{v}$.

2c

Examine if the vector field $\boldsymbol{v}$ has a potential ϕ, and if so find the potential.

2d

In this sub-problem we shall study the field for $z = 0$, notice that in this case the vector field is a plane field in the xy-plane.
Sketch a vector arrow plot (corresponding to *quiver* in Matlab or Python) for $\boldsymbol{v}$ in a region around the origin. Let the strength of the field be proportional to the length of the arrows.
Find all stagnation points (where $\boldsymbol{v} = \boldsymbol{0}$) and show them with the symbol $\bullet$.

2e

Find the circulation of $\boldsymbol{v}$ around the circle $\gamma : \{x^2 + y^2 = R^2, z = 0\}$ by direct computation. Let the circle be oriented such that we move from quadrant 1 to 2 to 3 to 4 in the xy-plane.
Control the answer by computing the circulation as a surface integral by employing a suitable integral theorem. What is the name of the integral theorem you use?

2f

Compute the integrated flux of $\boldsymbol{v}$ through the surface $S : \{x = 1, 0 \leq y \leq 1, -1 \leq z \leq 1\}$ oriented away from the origin.

2g

Find equations that represent the streamlines of the vector field $\boldsymbol{v}$. Describe with words how these streamlines behave for $z = 0$, for $z > 0$, and for $z < 0$.
Hint: The differential equations that must be solved may appear difficult since they do not immediately appear separable. They can still be brought over to separable form by introducing new variables r and θ such that $x = r\cos\theta$ and $y = r\sin\theta$.

END