UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in:	MEK 1100 — Vector calculus.
Day of examination:	Tuesday 21 March 2017.
Examination hours:	14:30 – 16:30.
This problem set consists of 2 pages.	
Appendices:	Two pages with formulas.
Permitted aids:	K. Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

There are 10 individual questions. Each individual question is given a score from 0 to 10 points (10 for complete answer, 0 for blank). Maximum possible score is 100 points. Check that you do not forget any of the questions.

Problem 1

A skier who moves with speed v is subjected to a drag force F due to air resistance and friction against the ground. For slow motion we shall assume F is approximately proportional to v with coefficient μ. For fast motion we shall assume F is approximately proportional to v^2 with coefficient ν. For an arbitrary speed we shall assume the following model

$$F = \mu v + \nu v^2. \tag{1}$$

The coefficients μ and ν are two free parameters which must be measured in carefully controlled experiments. We measure force in Newton (N) and speed in meters per second (m/s).

1a

Find the physical units of μ and ν.

1b

Scale equation (1) such that it is brought into non-dimensional form. Show how this can be done by introducing non-dimensional force F^* and non-dimensional speed v^* in such a way that there are no free parameters in the scaled and non-dimensional equation.

(Continued on page 2.)

Problem 2

A vector field is given in non-dimensional form by $\boldsymbol{v} = x\boldsymbol{i} - x\boldsymbol{j}$.

2a

Compute the divergence of $\boldsymbol{v}$.

2b

Compute the curl of $\boldsymbol{v}$.

2c

Determine if the vector field $\boldsymbol{v}$ has a potential ϕ, and in that case find ϕ.

2d

Determine if the field $\boldsymbol{v}$ has a stream function ψ, and in that case find ψ.

2e

Sketch a vector arrow plot (like *quiver* on the computer) for $\boldsymbol{v}$ near the origin. Let the magnitude of the field be proportional to the length of the arrows. Find all stagnation points (where $\boldsymbol{v} = \boldsymbol{0}$) and indicate them in the figure.

2f

Compute the stream lines of $\boldsymbol{v}$ and sketch them in a plot.

2g

Find the circulation of $\boldsymbol{v}$ around the boundary γ of the square $\Gamma : \{0 \leq x \leq 1, 0 \leq y \leq 1, z = 0\}$. Let the closed curve γ be oriented such that we go from the origin along the x-axis to $x = 1$, then to $x = y = 1$, then to the y-axis for $y = 1$ and then back to the origin. Compute this as a curve integral. Check the answer by computing the circulation as a surface integral by using a suitable integral theorem. Which integral theorem do you use?

2h

Compute the integrated flux of $\boldsymbol{v}$ out through the spherical shell $S : \{x^2 + y^2 + z^2 = 1\}$, oriented away from the origin.

END