

a) Euler:

$$\frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \nabla) \underline{u} = - \frac{\nabla p}{\rho} - \nabla \chi, \quad \underline{\omega} = \nabla \times \underline{u}$$

$$\nabla \cdot \underline{u} = 0$$

TA curl av Euler:

$$\frac{\partial \underline{\omega}}{\partial t} + \nabla \times (\underline{u} \cdot \nabla) \underline{u} = - \underbrace{\nabla \times \nabla \left(\frac{p}{\rho} + \chi \right)}_{=0}$$

$$\frac{\partial \underline{\omega}}{\partial t} + \nabla \times \left[(\nabla \times \underline{u}) \times \underline{u} + \frac{1}{2} \nabla \underline{u}^2 \right] = 0$$

$$\frac{\partial \underline{\omega}}{\partial t} + (\underline{u} \cdot \nabla) \underline{\omega} - (\underline{\omega} \cdot \nabla) \underline{u} + \underline{\omega} (\nabla \cdot \underline{u}) - \underline{u} (\nabla \cdot \underline{\omega}) + \frac{1}{2} \nabla \times \nabla \underline{u}^2 = 0$$

$$\Rightarrow \frac{\partial \underline{\omega}}{\partial t} + (\underline{u} \cdot \nabla) \underline{\omega} = \boxed{\frac{D \underline{\omega}}{Dt} = (\underline{\omega} \cdot \nabla) \underline{u}}$$

Hvis $\underline{\omega} = \underline{0}$ ved et tidspunkt er $\frac{\partial \underline{\omega}}{\partial t} = 0$.

Derved vil ikke vortubing kunne opstå.

Oppg 7

b) Friksjonskraft $F \sim C_D \rho u^2 A$

Effekt ved konstant hastighet: $P \sim \frac{F \cdot L}{t} \sim \frac{F \cdot L}{t} \sim F u$

$$\Rightarrow P = C_D \rho u^3 A$$

Vått areal per rom $A = V^{2/3}$

Vått areal for N rom $A = (NV)^{2/3}$

Effekt for N rom:

$$NP = C_D \rho u^3 (NV)^{2/3}$$

$$u = \frac{N^{1/9} P^{1/3}}{C_D^{1/2} \rho^{1/3} V^{2/9}}$$

c) Potentialstrømning ($\nabla \times \underline{u} = 0$) gir $\underline{u} = \nabla \phi$
Inkompressibilitet $\nabla \cdot \underline{u} = \nabla^2 \phi = 0$

$$\nabla^2 \phi = 0$$

Feltligning

$$\frac{\partial \phi}{\partial t} = -g\eta$$

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial y}$$

Lineariserte randubetingelser ved overflaten $y=0$

$$\frac{\partial \phi}{\partial y} = 0$$

Grensebetingelser ved havbunnen $y=-H$

Oppg 2

$$b) \quad 0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} + g \sin \alpha$$

$$u(z) = \frac{1}{2\nu} \frac{\partial p}{\partial x} z^2 + \frac{\rho g \sin \alpha}{\mu} z^2 + Az + B$$

$$\text{Hint: } \frac{\partial p}{\partial x} = \rho g \cos \alpha \frac{\partial h}{\partial x} \ll 1$$

$$u(z) = \frac{\rho g \sin \alpha}{2\mu} z^2 + Az + B$$

Gravitas betingelsen:

$$u(z=0) = B = 0$$

$$\mu \frac{\partial u(z=h)}{\partial z} = \rho g \sin \alpha h + A\mu = 0$$

$$A = - \frac{\rho g h \sin \alpha}{\mu}$$

$$u(z) = \frac{\rho g \sin \alpha}{2\mu} [z^2 - 2zh]$$

Kontinuitet:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

Integrene fra 0 til h:

$$\int_0^h \frac{\partial u}{\partial x} dz + \int_0^h \frac{\partial w}{\partial z} dz = \int_0^h \frac{\partial u}{\partial x} dz + w(h) - \underbrace{w(0)}_{=0} = 0$$

Randbedingung: $w(h) = \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x}$

Leibniz theorem: $\int_0^h \frac{\partial u}{\partial x} dz = \frac{\partial}{\partial x} \int_0^h u dz - u(z=h) \frac{\partial h}{\partial x}$

Einsetzen in Kontinuitätsgleichung:

$$\frac{\partial}{\partial x} \int_0^h u dz - \cancel{u(z=h) \frac{\partial h}{\partial x}} + \frac{\partial h}{\partial t} + \cancel{u(z=h) \frac{\partial h}{\partial x}} = 0$$

$$\frac{\partial h}{\partial t} = - \frac{\partial}{\partial x} \int_0^h \frac{\rho g \sin \alpha}{2\mu} [z^2 - 2hz] dz$$

$$= - \frac{\rho g \sin \alpha}{2\mu} \frac{\partial}{\partial x} \left[\frac{1}{3} z^3 - h z^2 \right]_0^h$$

$$= - \frac{\rho g \sin \alpha}{2\mu} \frac{\partial}{\partial x} \left[-\frac{2}{3} h^3 \right]$$

$$= \frac{\rho g \sin \alpha}{2\mu} \frac{\partial h^3}{\partial x}$$

$$= \frac{\rho g \sin \alpha}{\mu} h^2 \frac{\partial h}{\partial x}$$

Oppg 2

c) Slaking

$$\frac{h}{t} \sim \frac{\rho g \sin \alpha}{\mu} \frac{h^3}{x} \Rightarrow h = \left(\frac{\mu x}{\rho g \sin \alpha} \right)^{1/2} t^{-1/2}$$

Volumberegning 2D

$$A = \int_0^{x_N} h dx$$
$$= \int_0^{x_N} \left(\frac{\mu x}{\rho g \sin \alpha} \right)^{1/2} t^{-1/2} dx$$

$$= \left(\frac{\mu}{\rho g \sin \alpha} \right)^{1/2} \frac{2}{3} x_N^{3/2} t^{-1/2}$$

$$x_N(t) = \left(\frac{9A^2 \rho g \sin \alpha}{4\mu} \right)^{1/3} t^{1/3}$$

Oppg 3

$$\phi = U \left(r + \frac{a^2}{r} \right) \cos \Theta$$

$$a) \quad u_r = \frac{\partial \phi}{\partial r}$$

$$= U \left(1 - \frac{a^2}{r^2} \right) \cos \Theta$$

$$u_\Theta = \frac{1}{r} \frac{\partial \phi}{\partial \Theta}$$

$$= -U \left(1 + \frac{a^2}{r^2} \right) \sin \Theta$$

Vis at strømmingene er uforstyrret ved $r = \infty$

Test $\Theta = 0, \pi/2$

$$\begin{aligned} \underline{u}(r=\infty, \Theta=0) &= U(1-0) \cdot 1 \leq r - U(1+0) \cdot 0 \leq 0 \\ &= \underline{U \leq r} \end{aligned}$$

$$\begin{aligned} \underline{u}(r=\infty, \Theta=\pi/2) &= U(1-0) \cdot 0 \leq r - U(1+0) \cdot 1 \leq 0 \\ &= \underline{-U \leq 0} \end{aligned}$$

ok!

Oppg 3

b) Grensebetingelse på sylindervegg $r=a$:

Ingen gjennomstrømming: $u_r(r=a, \theta) = 0$

Slip betingelse: $u_\theta(r=a, \theta) \neq 0$

$$u_r(r=a, \theta) = U \left(1 - \frac{a^2}{r^2} \right) \cos \theta$$

$$= \underline{0}$$

$$u_\theta(r=a, \theta) = -U \left(1 + \frac{a^2}{r^2} \right) \sin \theta$$

$$= \underline{-2U \sin \theta}$$

Opp. Vi har to stagnasjonspunkt ved $r=a$ og $\theta=0, \pi$

Üppg 3

c) Cauchy-Riemann

$$u_r = \frac{\partial \phi}{\partial r} = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad (\text{I}), \quad u_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = - \frac{\partial \psi}{\partial r} \quad (\text{II})$$

$$\begin{aligned} (\text{I}): \quad \int \frac{\partial \psi}{\partial \theta} d\theta &= \int r u_r d\theta \\ &= \int U \left(r - \frac{a^2}{r} \right) \cos \theta d\theta \end{aligned}$$

$$\psi = U \left(r - \frac{a^2}{r} \right) \sin \theta + \psi_0$$

$$\begin{aligned} (\text{II}) \quad \int \frac{\partial \psi}{\partial r} dr &= - \int u_\theta dr \\ &= \int U \left(1 + \frac{a^2}{r^2} \right) \sin \theta dr \end{aligned}$$

$$\psi = U \left(r - \frac{a^2}{r} \right) \sin \theta + \psi_1$$

$$\Rightarrow \quad \psi = U \left(r - \frac{a^2}{r} \right) \sin \theta$$

Komplexe potentielle $w(z) = \phi + i\psi$, $z = re^{i\theta}$

$$w = U \left(r + \frac{a^2}{r} \right) \cos \theta + i U \left(r - \frac{a^2}{r} \right) \sin \theta$$

$$= U r (\cos \theta + i \sin \theta) + U \frac{a^2}{r} (\cos \theta - i \sin \theta)$$

$$= U r e^{i\theta} + U \frac{a^2}{r} e^{-i\theta}$$

$$= \underline{U \left(z + \frac{a^2}{z} \right)}$$

Oppg 3

d) Finn trykk på sylindren uha Bernoulli:

$$p + \frac{1}{2} \rho \underline{u^2} = \underbrace{\text{konstant}}_{C_0}$$

$$p(n=a) = C_0 - \frac{1}{2} \rho \underline{u^2(n=a)}$$

$$= C_0 - \frac{1}{2} \rho (\cancel{u_n^2(n=a)} + u_0^2(n=a))$$

$= 0$

$$= C_0 - 2 \rho U^2 \sin^2 \theta$$

Dnag

$$F_x = \int_0^{2\pi} (C_0 - 2 \rho U^2 \sin^2 \theta) a \cos \theta d\theta$$

$$= \left[C_0 a \sin \theta - 2 \rho U^2 a \sin^3 \theta \right]_0^{2\pi}$$

$$= \underline{0}$$

Lift

$$F_y = - \int_0^{2\pi} (C_0 - 2 \rho U^2 \sin^2 \theta) a \sin \theta d\theta$$

$$= \left[C_0 a \cos \theta - \frac{1}{6} \rho U^2 a \cos(3\theta) + \frac{3}{2} \rho U^2 a \cos(\theta) \right]_0^{2\pi}$$

$$= \underline{0}$$

Ideell fluid ; ingen friksjon.