

UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Exam in: MEK 4100 — Mathematical methods in mechanics.

Day of examination: Thursday 15 June 2023.

Examination hours: 9:00–13:00.

This problem set consists of 3 pages.

Appendices: Formula sheet.

Permitted aids: K. Rottmann: Matematisk Formelsamling,
approved calculator.

Please make sure that your copy of the problem set is
complete before you attempt to answer anything.

Problem 1 (weight 20%)

Gravity-capillary waves with wavenumber k and angular frequency ω propagate on a current with speed U and also depend on the acceleration of gravity g and the surface tension σ and the water density ρ .

Recall that the wavenumber k has dimension inverse of length, the angular frequency ω has dimension inverse of time, the surface tension σ has dimension force per length, and force has dimension mass times acceleration.

1a (weight 10%)

Determine the minimal number of dimensionless parameters that are needed to describe these waves. Find a possible set of such dimensionless parameters.

1b (weight 10%)

The relation between ω and k is called the dispersion relation of the waves. Find the most general expression for the dispersion relation without assuming any further knowledge of the governing hydrodynamic equations.

Problem 2 (weight 10%)

Solve the Volterra integral equation

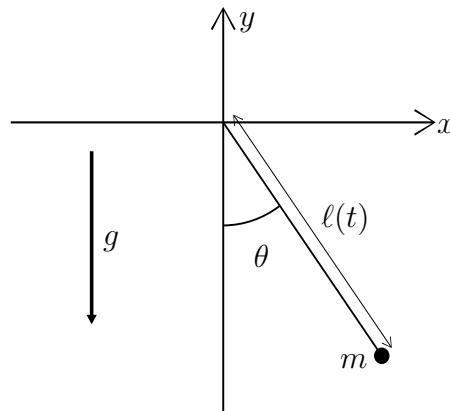
$$y(x) = \int_0^x (x-s)y(s) \, ds + 1$$

Hint: The integral equation may correspond to an initial value problem?

(Continued on page 2.)

Problem 3 (weight 10%)

A pendulum with variable length $\ell(t)$ has angular displacement $\theta(t)$ with respect to the equilibrium downward position. The variable length $\ell(t)$ is determined by some external influence. We want to derive an equation for the resulting angular displacement $\theta(t)$.



Find the Lagrangian and the Lagrange equation.

Problem 4 (weight 20%)

Find the first two terms of a perturbation solution of the algebraic equation

$$\sqrt{x} = \cos(\epsilon x)$$

where $0 < \epsilon \ll 1$.

Hint: One possible approach is to do the substitution $t = \sqrt{x}$.

Problem 5 (weight 20%)

Use the WKB method to solve the initial value problem

$$\epsilon y'' + q(x)(y' + y) = 0, \quad y(0) = 1, \quad y'(0) = \epsilon^{-1}$$

where $q(0) \neq 0$ and $0 < \epsilon \ll 1$.

Hint: Start by employing the transformation $y = \exp(S)$.

Be careful to identify two solutions for S corresponding to the two fundamental solutions of the equation.

(Continued on page 3.)

Problem 6 (weight 20%)

Use a perturbation approach to solve the initial value problem

$$y'' + y = \epsilon y^3, \quad y(0) = 0, \quad y'(0) = 1$$

where $0 < \epsilon \ll 1$.

Find the solution up to the first two orders of ϵ .

THE END