

boundary layer, and compressible (Chap. 7) flows. For nonboundary-layer flows, the use of computational fluid dynamics (CFD) for both laminar and turbulent flow is an increasingly powerful tool.

PROBLEMS

- 3-1. Reconsider the problem of Couette flow between parallel plates, Fig. 3-1, for a power-law nonnewtonian fluid, $\tau_{xy} = K(du/dy)^n$, where $n \neq 1$. Assuming constant pressure and temperature, solve for the velocity distribution $u(y)$ between the plates of (a) $n < 1$ and (b) $n > 1$, and compare with the newtonian solution, Eq. (3-6). Comment on the results.
- 3-2. Consider the axial Couette flow of Fig. 3-3 with both cylinders moving. Find the velocity distribution $u(r)$ and plot it for (a) $U_1 = U_0$, (b) $U_1 = -U_0$, and (c) $U_1 = 2U_0$. Comment on the results.
- 3-3. Consider the axial Couette flow of Fig. 3-3 with the inner cylinder moving at speed U_0 and the outer cylinder fixed. Solve for the temperature distribution $T(r)$ in the fluid if the inner and outer cylinder walls are at temperatures T_0 and T_1 , respectively.

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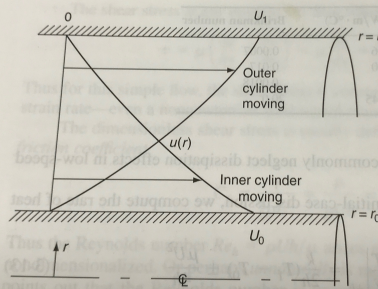


FIGURE 3-3
Axial annular Couette flow between concentric moving cylinders. The velocity distributions are plotted from Eqs. (3-18) and (3-19).

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- 3-4. A long thin rod of radius R is pulled axially at speed U through an infinite expanse of still fluid. Solve the Navier-Stokes equation for the velocity distribution $u(r)$ in the fluid and comment on a possible paradox.
- 3-5. A circular cylinder of radius R is rotating at steady angular rate ω in an infinite fluid of constant ρ and μ . Assuming purely circular streamlines, find the velocity and pressure distribution in the fluid and compare with the flow field of an inviscid "potential" vortex.
- 3-6. Assuming that the velocity distribution between rotating concentric cylinders is known from Eq. (3-22), find the pressure distribution $p(r)$ if the pressure is p_0 at the inner cylinder.
- 3-7. An open U tube of radius r_0 filled with a length L of a viscous fluid is displaced from rest and oscillates with amplitude $X(t)$, as shown in Fig. P3-7. Show with a one-dimensional integral analysis that the governing equation for $X(t)$ is

$$\frac{d^2 X}{dt^2} + \frac{C_f}{r_0} \left| \frac{dX}{dt} \right| \frac{dX}{dt} + \frac{2g}{L} X = 0$$

Sketch velocity profiles for a few representative times, including $t = 0$.

- 3-15. Consider a wide liquid film of constant thickness h flowing steadily due to gravity down an inclined plane at angle θ , as shown in Fig. P3-15. The atmosphere exerts constant pressure and negligible shear on the free surface. Show that the velocity distribution is given by

$$u = \frac{\rho g \sin \theta}{2\mu} y(2h - y)$$

and that the volume flow rate per unit width is $Q = \rho g h^3 \sin \theta / 3\mu$. Compare this result with flow between parallel plates, Eqs. (3-44) and (3-45).

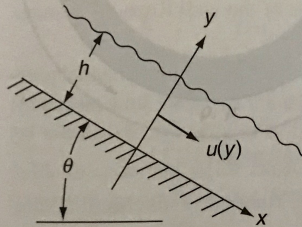


FIGURE P3-15

- 3-16. Consider a film of liquid draining at volume flow rate Q down the outside of a vertical rod of radius a , as shown in Fig. P3-16. Some distance down the rod, a fully developed region is reached where fluid shear balances gravity and the film thickness remains constant. Assuming incompressible laminar flow and negligible shear interaction with the atmosphere, find an expression for $v_z(r)$ and a relation between Q and film radius b .

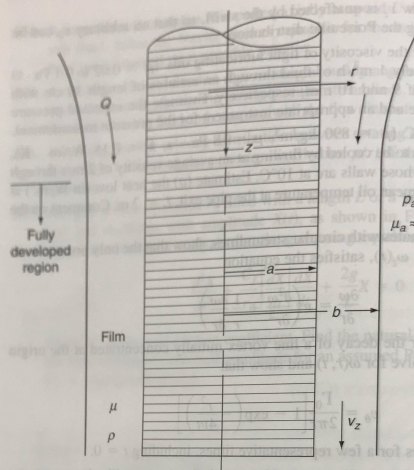


FIGURE P3-16

- 3-17. By extension of Prob. 3-15, consider a *double layer* of immiscible fluids 1 and 2, flowing steadily down an inclined plane, as in Fig. P3-17. The atmosphere exerts no shear stress on the surface and is at constant pressure. Find the laminar velocity distribution in the two layers.

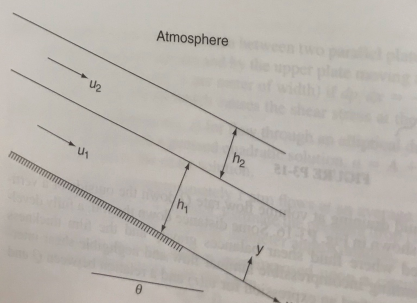


FIGURE P3-17