

Exercises for 31/8-2022

Exercise 1

Dissipation in viscous fluid flow be thought of as the loss of kinetic energy over time. Starting from the Navier-Stokes momentum equation for an incompressible fluid, derive the expression for the viscous dissipation in an infinite body of fluid as:

$$\frac{\partial E_{kinetic}}{\partial t} = -\frac{1}{2}\mu \int \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 dV \quad (1)$$

with μ the viscosity and V the fluid volume. What does this tell us about the coefficient of viscosity?

Exercise 2

Non-dimensionalize the Navier-Stokes momentum equation using some characteristic scales (*e.g.* $x_i^* = x_i L$ with the asterix indicating non-dimensional variables and L being the characteristic length of the system) and define the non-dimensional parameters that appear in the equation. What are these parameters called, what do they express, and describe the flow in the limits of these parameters.

Exercise 3

Write up the boundary conditions that are applicable at a:

- solid-liquid interface
- liquid-liquid interface
- liquid-gas interface

You should also justify why the ones you choose are appropriate.

Exercise 4

Consider the steady form of the incompressible Navier-Stokes momentum equation, *i.e.* $u \neq u(t)$. Use dimensional analysis to find the drag force on an object of characteristic length L in the low and high Reynolds number limit ($Re = \rho UL/\mu$, with ρ being the fluid density and U the characteristic velocity).