

MEK4300/9300
Mandatory exercise 1
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Exercise 1

Dissipation in viscous fluid flow be thought of as the loss of kinetic energy over time. Starting from the Navier-Stokes momentum equation for an incompressible fluid, derive the expression for the viscous dissipation in an infinite body of fluid as:

$$\frac{\partial E_{kinetic}}{\partial t} = -\frac{1}{2}\mu \int \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 dV \quad (1)$$

with μ the viscosity and V the fluid volume. What does this tell us about the coefficient of viscosity?

Exercise 2

Consider a fluid initially at rest between two infinite parallel plates located at $y = 0$ and $y = h$ respectively. At $t = 0$ a pressure gradient is applied in the x -direction forcing the fluid to flow. We assume that the flow is invariant in the z -direction making the flow 2-dimensional and that the no-slip condition can be applied at both plates.

1. Set up the governing flow equation including initial and boundary conditions
2. Solve the flow equation for a constant pressure gradient, *i.e.* $dp/dx = \text{constant}$.
3. Calculate the force exerted by the fluid on the plates.

Exercise 3

We now want to study our problem numerically and we therefore need to restrict our domain to a finite size. We do this by imposing open horizontal boundaries located at $x = 0$ and $x = L$.

1. Using the method of finite differences, find a suitable scheme to discretize the flow equation and write up the discrete formulation of the flow equation.
2. Find the stability criterion for your scheme
3. Implement the scheme numerically using *e.g.* Python, matlab, etc.
4. Compare the flow profiles from Exercise 2b and the force estimate for Exercise 2c with your numerical results.

Note: You should take care in choosing appropriate numerical values for h and L when solving the equation.

Exercise 4

Redo Exercise 2b and 3d using the real part of an oscillating pressure gradient $dp/dx = -P_0 e^{i\omega t}$ with i the imaginary unit and P_0 and ω being the oscillation amplitude and frequency, respectively.