

Exercises for 12/9-2022

Exercise 1

Consider the flow between two porous parallel plates located at $y = -h$ and $y = h$. The flow is generated by a constant pressure gradient and we assume that the wall porosity is such that it generates a uniform vertical crossflow of constant velocity $v = v_c$. Continuity then yields $u = u(y)$ which gives a momentum equation

$$\rho v_c \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}. \quad (1)$$

In the flow direction we retain the no-slip condition $u(h) = u(-h) = 0$. Find the solution to the flow field $u(y)$ and write it as a function of the Reynolds number $Re = \rho v_c h / \mu$. Plot the solution for a select number of values for Re . What are the limits of this solution?

Exercise 2

Consider an infinitely long plate with an unbound fluid above it. The plate is suddenly accelerated to a constant velocity V such that the fluid momentum equation with boundary and initial conditions are expressed as

$$\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}, \quad (2)$$

$$\begin{aligned} u(y, t = 0) &= 0, \\ u(y = 0, t) &= U, \\ u(y \rightarrow \infty, t) &= 0. \end{aligned} \quad (3)$$

Introduce a similarity variable $\eta = y/(2\sqrt{\nu t})$ and find the similarity solution $f(\eta) = u/U$. Plot u as a function of y for different values of t and compare with f plotted as a function of η . What is the force exerted on the plate?

Exercise 3

Redo Problem 2 for an arbitrary wall motion, *i.e.* $u(y = 0, t) = g(t)$.

Exercise 4

Consider the same problem as given in Problem 2, but now with an suddenly applied shear stress such that the no-slip condition in Problem 2 is replaced by

$$\frac{\partial u}{\partial y} = K, \quad \text{at } y = 0. \quad (4)$$

Hint: It might be necessary to consider a more general similarity solution $u = g(t)f(\eta)$. A good first guess for $g(t)$ is a power-law solution, *i.e.* $g(t) = t^\alpha$ with *e.g.* $\alpha = 1/2$.