MEK4350, fall 2014
Exercises 6

We have learnt that if X is a random variable, and $F(x)$ is the cumulative distribution function for X, then the probability density function is given by

$$f(x) = \frac{\mathrm{d}F}{\mathrm{d}x},$$

the moden is the value for x where $f(x)$ achieves its maximum, the median is the value for x where $F(x) = 0.5$, the mean (or expected value) is

$$\mu = E[X] = \int_{-\infty}^{\infty} x f(x)\, dx,$$

the variance is

$$\sigma^2 = E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx,$$

the standard deviation is

$$\sigma = \sqrt{\sigma^2}$$

the skewness is

$$\gamma = \frac{E[(X-\mu)^3]}{\sigma^3}$$

and the kurtosis is

$$\kappa = \frac{E[(X-\mu)^4]}{\sigma^4}$$

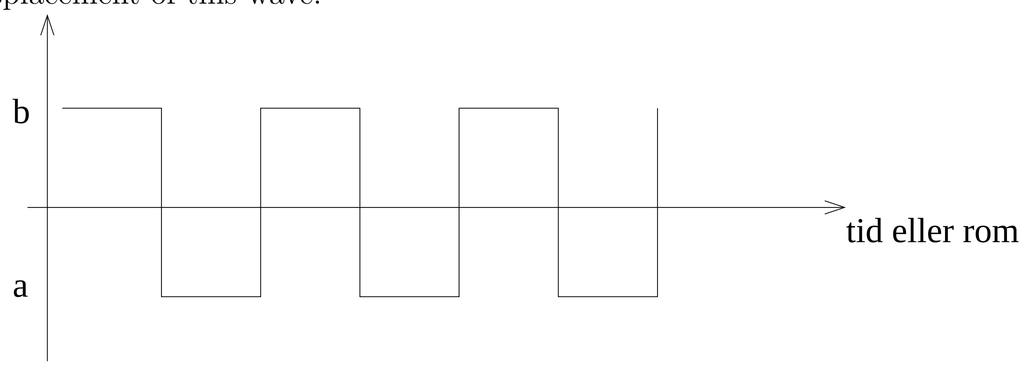
We shall compute all of these cuantities for some important probability distributions. Some of these distributions are so important that we should know them by heart. Notice in particular which of these have kurtosis less than, equal to or greater than 3.

Problem 1: Toss heads and tails.

Let the outcome "heads" have value a and the outcome "tails" have value b.

$$f(x) = \frac{\delta(x-a) + \delta(x-b)}{2}$$

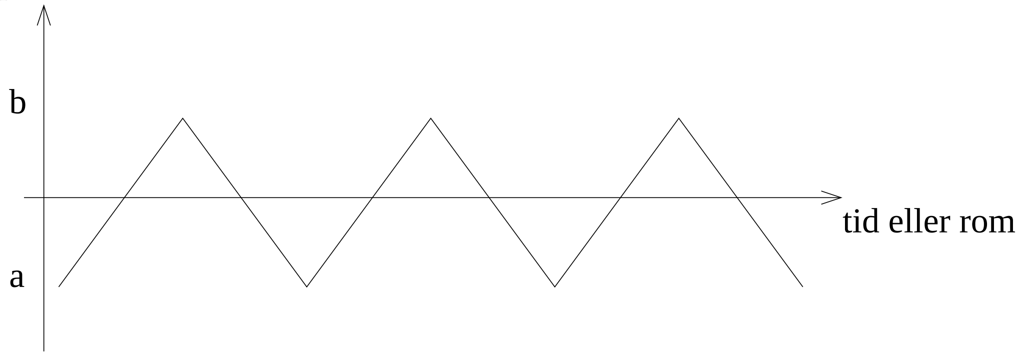
Convince yourself that this is the probability density distribution for the surface displacement of this wave:



Problem 2: Uniform distribution between a and b, for $a < b$

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for} \quad a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Convince yourself that this is the probability density function for the surface displacement of this wave:



Problem 3: Normal or Gaussian distribution $N(\mu, \sigma)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Problem 4: Exponential distribution

$$f(x) = \lambda e^{-\lambda x} H(x)$$

Notice: We have used the Heaviside step function to write the formula in a compact way, such that we have $f(x) = 0$ for negative x.

Problem 5: Rayleigh distribution

$$f(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} H(x)$$

Note 1: We have used the Heaviside step function to write the formula in a compact way.

Note 2: In the standard expression for the Rayleigh distribution the paramter σ is commonly used, however it is not the standard deviation of X!