

MEK4350, fall 2014
Exercises 7

Moments and characteristic function

The moments of a random variable X are

$$m_n = E[X^n].$$

The characteristic function of X is

$$\phi(k) = E[e^{ikX}].$$

Problem 1

Express the variance σ^2 , skewness γ and kurtosis κ exclusively by the moments m_1 , m_2 , m_3 and m_4 .

Problem 2

Show that the moment m_n can be found by computing the n -th derivative of $\phi(k)$ and evaluate the result for $k = 0$.

Hint: You may use the power series expansion of the exponential function.

Problem 3

Let X be uniformly distributed between 0 and 2π . Compute the characteristic function of X . Compute the variance, skewness and kurtosis by expanding and/or differentiating the characteristic function. Compare the result with Problem 2 in Exercises 6.

Problem 4

Let X be Gaussian distributed with mean μ and variance σ^2 . Given that the characteristic function of X is $\phi(k) = e^{i\mu k - \frac{\sigma^2 k^2}{2}}$, compute the skewness and kurtosis of X by expanding and/or differentiating the characteristic function. Compare the result with Problem 3 in Exercises 6.