20th September, 2022

MEK4350

Mandatory assignment 1 of 2

Submission deadline

Thursday 6th October 2022, 14:30 on Canvas canvas.uio.no.

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

NB 1: You don't have to read any of the papers referred to in these problems. Please do not spend time reading those papers! You should only look at the figures in McLean *et al.* (1981), but do not try to understand how they computed those figures! The reference can be found on Leganto.

NB 2: The problems suggested in the paragraphs beginning with "If you want to do more . . . " are completely optional!

Problem 1. Show that there can be quintet resonance for gravity waves with three waves that are identical $\boldsymbol{k}_1 = \boldsymbol{k}_2 = \boldsymbol{k}_3 = k(1,0)$ and two waves $\boldsymbol{k}_4 = k(1+p,q)$ and $\boldsymbol{k}_5 = k(2-p,-q)$ that do not have to be parallel to the first three. Show that you get a relationship of the type shown in equation (6) in the paper McLean *et al.* (1981).

Plot the curve (p,q) for quintet resonance together with the "figure 8" of Phillips (1960) for quartet resonance, do this for the special case of infinite depth. Show that you reproduce the two curves closest to the origin shown in figure 2 of McLean *et al.* (1981).

If you want to do more on quintet resonance, show how the curve you found above is deformed as the depth becomes smaller, similar to figure 7 in the lecture notes `Nonlinear.pdf`.

Problem 2. The modulational instability known as the Benjamin–Feir instability (after Benjamin & Feir, 1967) happens in a domain delimited on one side by the "figure 8" of Phillips (1960), and on the other side by a different curve. This is seen as the shaded area near the origin in figure 1 of McLean *et al.* (1981). The domain of instability found from the NLS equation gives a leading order approximation of the exact domain found by McLean *et al.* (1981).

Compute the growth rate for sideband instability of the NLS equation in two horizontal dimensions. Compute the location of the most unstable perturbations. Show that the "figure 8" of Phillips (1960) is tangent to the pair of straight lines crossing at the origin found in the analysis of the NLS equation. Make a contour plot of the unstable growth rate of the Benjamin–Feir instability according to the NLS equation. In the same plot show clearly the location of the most unstable instability you computed above. In the same plot also show the "figure 8" of Phillips (1960).

If you want to do more on BF-instability, take into account that modulation of ocean waves tends to happen on faster scales $(t_{\frac{1}{2}}, x_{\frac{1}{2}}, y_{\frac{1}{2}}) = \epsilon^{\frac{1}{2}}(t,x,y)$ and therefore the cubic nonlinear term should balance fourth derivatives with respect to x and y. This will modify the shape of the instability region predicted by the modified NLS equation. Does this bring us closer to the exact result in McLean *et al.* (1981)?

Bibliography

Benjamin, T. B. & Feir, J. E. 1967 The disintegration of wave trains on deep water. *J. Fluid Mech.* **27**, 417–430.

McLean, J. W., Ma, Y. C., Martin, D. U., Saffman, P. G. & Yuen, H. C. 1981 Three-dimensional instability of finite-amplitude water waves. *Phys. Rev. Lett.* **46**, 817–820.

Phillips, O. M. 1960 On the dynamics of unsteady gravity waves of finite amplitude. Part 1. The elementary interactions. *J. Fluid Mech.* **9**, 193–217.