MEK 4450
Stratified flow models
Fundamentals of slug flow

Lecture notes
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Stratified flow models
Introduction

• The different flow require different terms (closure relations) in momentum equations for
  • Wall and interfacial friction factors
  • Dispersion of phases as drops and bubbles
  • Momentum transfer between phases due to phase change

• The last point is closely connected with thermodynamics and will not be treated here

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Pure stratified flow – volume fractions

Void fraction: \( \alpha = \frac{A_g}{A} \)

Oil holdup: \( \beta_o = \frac{A_o}{A} \)

Water holdup: \( \beta_w = \frac{A_w}{A} \)

Total holdup: \( \beta = \beta_o + \beta_w \)
Shear stresses in pipe flow

- Single phase flow:

\[ \tau = \frac{1}{8} \lambda \rho U^2 \]

\[ \frac{\partial p}{\partial z} = -\frac{\tau S}{A} = -\frac{1}{2} \frac{\lambda \rho U^2}{2} \frac{S}{4A} = -\frac{1}{2} \frac{\lambda \rho U^2}{D} \]

- Laminar flow: Hagen-Poiseuille
  \[ \lambda = \frac{64}{\text{Re}} \quad \text{Re} = \frac{UD\rho}{\mu} \]

- Turbulent flow: Håland
  (Explicit approximation to Colebrook-White formula)

\[ \frac{1}{\sqrt{\lambda}} = -1.8 \log_{10} \left[ \frac{6.9}{\text{Re}} + \left( \frac{\varepsilon}{3.7D} \right)^{1.11} \right] \]
Shear stresses in channel flow

- Pressure drop in channel flow calculated using hydraulic diameter concept
- Hydraulic diameter = Diameter of "equivalent" circular pipe
- Good approximation in most cases
- Not for geometries with narrow corners (i.e. triangle)
- Pressure drop:

\[
\frac{\partial p}{\partial z} = -\frac{1}{2} \lambda \rho U^2 \frac{1}{D_H}
\]

\[
D_H \equiv \frac{4A}{S}
\]

\[
\lambda = \lambda(\text{Re}, \varepsilon / D_H)
\]

\[
\text{Re} = \frac{UD_H \rho}{\mu}
\]
Hydraulic diameters: Traditional method

• The gas is regarded as flowing in a closed channel
• The liquid is regarded as flowing in an open channel
• For three-phase gas-oil-water, the choices are less obvious and demonstrate limitations of the concept
Gas wall friction factor

- Simplest approach: Hydraulic diameter concept with closed channel
- Gas velocity profile is generally asymmetrical in wavy flow, leading to increase of shear stresses
- Friction factor can be corrected for waves
Liquid wall friction factor

- Smooth stratified flow (no waves): Hydraulic diameter approach gives good results
- Wavy stratified flow: Hydraulic diameter approach gives poor results because velocity profile is modified by waves (Espedal 1998)
- Several correlations proposed in literature
- Biberg (2007) proposed a model computing friction factors by matching turbulent velocity profiles in both phases at interface
**Interfacial friction factor**

- Many different correlations in literature
- Most based on low pressure air/water experiments
- Poor extrapolation properties to high pressure
- The model of Biberg (2007) gives the interfacial shear stress from the continuity of gas and liquid velocities at the interface
Real stratified flow

Continuous gas
Gas bubbles in oil
Gas bubbles in water
Continuous oil
Oil drops in gas
Oil drops in water
Continuous water
Water drops in gas
Water drops in oil

\begin{align*}
\alpha_g & \\
\alpha_o & \\
\alpha_w & \\
\beta_{oc} & \\
\beta_{od} & \\
\beta_{wd} & \\
\beta_{wc} & \\
\gamma_o & \\
\gamma_w & \\
\end{align*}
Dispersions in stratified flow

- drop transport (entrainment) in the gas
- Gas bubbles in the liquid layer
- Oil-water dispersions
Drop transport in gas

- At high gas flow rates, drops are torn off the interface and flow along with the gas \((\text{entrainment})\)
- Equilibrium is achieved between drop entrainment from the liquid and \(\text{deposition}\) of drops to the liquid
- More than 50% of the volume flow of liquid can be in the form of drops
- There is some correlation between the drop entrainment and the \(\text{Weber number}\)

\[ We = \frac{\rho_g U_g^2 D_{hg}}{\sigma} \]
Gas bubbles in the liquid

- Gas is entrained as bubbles into a liquid layer at the gas-liquid interface
- Known from hydraulic engineering (rivers, dams)
- Occurs as result of turbulence and breaking waves
- Equilibrium gas fraction determined by available kinetic energy
- Significant gas entrainment occurs above a geometry dependent critical Froude number \( U = \sqrt{gH} \)
Oil-water dispersions

- Oil can be dispersed into water as drops, or water into oil
- Dispersions can be stabilized by shear or chemicals (surfactants)
- A dispersion of small drops stabilized by surfactants is called an emulsion
- Important for pressure drop
  - Dispersed drops increase pressure drop in most cases because drops behave like solid particles
  - In some cases dispersed drops can decrease pressure drop because drops are deformed by shear stress
  - Phase inversion (Transition between water drops in oil and oil drops in water) can give large pressure drop
Oil-water dispersions

- Very different flow regime and pressure drop as function of water fraction (water cut) for different oils

- **Surface chemistry** affects drop breakup and coalescence – interfaces can be rigid or flexible

Two-phase oil-water flow
Umix = 1.75 m/s, From Utvik et al (1999)

![Graph showing the relationship between water cut and relative pressure gradient for different oil-water dispersions.](image)

ST = stratified, DO = *Dispersed oil continuous*, DW = *Dispersed water continuous*
Emulsion viscosity

- In oil-water dispersions the **apparent viscosity** increases dramatically towards **phase inversion**
- Einstein developed formula for the apparent viscosity for a suspension of hard spheres in liquid:
  \[
  \frac{\mu_{\text{mixture}}}{\mu_{\text{pure}}} = 1.0 + \frac{5}{2} \Phi
  \]
  \[\Phi = \text{drop concentration}\]
- Later workers extended Einstein’s formula to more realistic systems, notably Pal and Rhodes (1999)
Summary

• Stratified models need to include models for:
  • Flow geometry
  • Wall and interfacial friction factors
  • Drops in gas phase
  • Gas bubbles in liquid layer
  • Oil drops in water and water drops in oil

• Hydraulic diameters are often used for modelling friction factors but fails for wavy flow

• Friction factors can be modelled more accurately from detailed models of velocity profiles

• Drops and bubbles influenced by surface chemistry
Fundamentals of slug flow
What is slug flow?

- Long bubbles (*Taylor bubbles*) alternate with more or less aerated liquid plugs, called *slugs*
  
  ![Diagram of slug flow](image)
  
  - The slug front (bubble tail) acts like a *hydraulic jump*
  - Gas bubbles are *entrained* into the slug front and transported backward relative to the front
  - The liquid below the bubble can also contain bubbles
What is slug flow (II)

- Mechanisms in slug flow
- From Hale et al. (2000)

![Diagram of slug flow mechanisms](image-url)

- Gas entrained by slug body.
- Gas released from slug body.
- Liquid film "scooped up" by faster moving slug body.
- Aerated liquid "shed" from faster moving slug.
- Gas released from film.
What is slug flow (III)

- Behind the slug front there is normally a *mixing zone* with strong turbulence
  - Increased wall shear stress (friction)
  - Gas entrainment
- The gas is nearly homogeneously distributed in the mixing zone
- The gas is rising towards the top of the pipe behind the mixing zone at low velocities
Severe slugging (terrain slugging)

- Occurs in dip geometry at low flow velocity
- Liquid slows down and blocks gas from passing the bend
- Pressure builds up and starts pushing out liquid
- Long bubble expands and blows rapidly out on top
- Cycle repeats
- Gives large pressure and flow oscillations, disturbing process
- Can be controlled by *choking* (valve with small opening)

Drawing: ABB
Horizontal and vertical slug flow

- Long, near horizontal pipes:
  - Relatively long slugs (> 30 diameters)
- Vertical pipes:
  - Shorter slugs (typically < 20 diameters)
  - Falling liquid around the Taylor bubble
- In vertical flow Taylor bubbles are only stable for pipes of diameter less than about 10 cm
- In larger diameter pipes we normally get chaotic flow (churn flow) instead of slug flow
- Slugs developed in a long pipe can survive a short riser, but will die out in a long, large diameter riser
Flow regime transitions

- The most important flow regime transition in pipelines is from stratified flow to slug flow.
- Two conditions must be fulfilled for slug flow to exist:
  - Stratified flow must be unstable (Kelvin-Helmholtz instability).
  - Slugs that are formed must be able to grow (Minimum slip).
- The Kelvin-Helmholtz criterion tells that the stratified flow region gets smaller with increasing pressure.
- Experimental data show that the slug flow region also gets smaller with increasing pressure.
- For high pressure, we get a region of large wave flow in between stratified and slug.
In between

$U_{SL}$

slugs are stable

neither is stable

stratified flow is stable

$U_{SG}$
Slug formation

- Complex phenomenon, not fully understood
- Studies at Imperial College, Hale et al. (2001)
- "Slug precursor" formed on top of long wave formed after previous slug
- Most "slug precursors" collapse, while a few survive and grow into long slugs
- Other mechanisms as well

Figure 11. Slug formation mechanism of Taitel and Dukler (1977).
Slug growth

- Most new, short slugs collapse because there isn’t enough liquid ahead of them to grow on.
- Liquid left by a collapsed slug is picked up by the next one.
- Slug frequency gradually decaying along the pipe.
- Terrain effects from ups and downs important in long pipelines.
History of slug flow modelling

- Theoretical modelling since the 1970s
- Traditional concept: *Unit Cell Model*
  - Pipe discretized into control volumes
  - Fully developed flow assumed in each control volume
  - Development of each individual slug ignored
- The unit cell model cannot predict slug length
- More modern approach: *Slug tracking*, where slugs are tracked from they are formed until they vanish
- Main challenge: Model *slug formation* and growth
Unit cell model for slug flow

- Most common concept for slug flow modelling
  - Infinite train of identical slug and Taylor bubbles
  - Fully developed dispersed bubble flow assumed in slug
    - Sometimes extended to include increased friction behind the slug front
  - Fully developed stratified/annular flow assumed in Taylor bubble zone
    - Sometimes extended to include developing flow (slug tail profile)
- Has also been extended to gas-oil-water 3-phase flow
Unit cell model: Assumptions

• The flow is assumed steady and periodical in a frame of reference moving with the pattern velocity $U_B$
• In other words the flow is assumed locally fully developed
• The slug front and the bubble nose are assumed to have the same velocity $U_B$
Slug collapse in downward flow

- For low mixture velocities we normally get slug flow uphill and stratified flow downhill
- The unit cell model assumes *local equilibrium* independent of upstream and downstream conditions
- The model will therefore often predict that slugs vanish immediately when they reach a hilltop
- In reality slugs can often survive for a considerable distance in a downhill if slug collapse is slow
- Slug tracking or similar techniques are necessary to study how slugs develop in a hilly terrain pipeline
Unit cell model: Continuity (I)

- Total volume flux (mixture velocity) is \( U_M = U_{SG} + U_{SL} \)
- Total mass flow is constant along the pipe (in a stationary frame of reference)
- If we assume constant densities, the mixture velocity is also constant along the pipe
- Volume flux in the slug: \( \alpha_S U_{GS} + (1 - \alpha_S) U_{LS} = U_M \)
- Volume flux in the bubble: \( \alpha_B U_{GB} + (1 - \alpha_B) U_{LB} = U_M \)
Unit cell model: Continuity (II)

- In a frame of reference moving with the fronts, continuity of each phase across the bubble nose gives

\[ \alpha_s (U_{GS} - U_B) = \alpha_B (U_{GB} - U_B) \]

\[ (1 - \alpha_s) (U_{LS} - U_B) = (1 - \alpha_B) (U_{LB} - U_B) \]
Closure relations for the unit cell

- **Bubble nose velocity**
- Void fraction (gas volume fraction) in slugs
- Velocity difference between gas and liquid in slugs
- Friction factors in bubble zone and in slug
- *Extra pressure drop* behind slug front
The bubble nose velocity

- The bubble nose velocity is approximately linear in the mixture velocity: \( U_B = C_0 U_M + U_0 \)
- The *distribution coefficient* \( C_0 \) is a function of the velocity profile in the slug
  - \( C_0 \approx 2 \) for laminar flow in the slug
  - \( C_0 \approx 1.2 \) for fully turbulent flow in the slug
- Smooth transition between laminar and turbulent flow
- The *drift velocity* \( U_0 \) is a function of inclination angle, densities, liquid viscosity and pipe diameter
The gas fraction in slugs

- The gas fraction in the slug depends on
  - Velocity distribution, pipe diameter, inclination, densities, viscosities, surface tension, surface rheology
  - The surface rheology and its influence is little known
- Gas fraction in slugs is complex and difficult to model
  - Empirical correlations mostly used. Typically $\alpha_S = \alpha_S(U_M, \theta)$
  - Some simple mechanistic models exist
    - Predicts gas entrainment at front and the gas distribution in the slug
    - Typically poor extrapolation properties
The gas distribution in slugs

- Gas is entrained into the slug at the front.
- The gas entrainment can be increased by gas in the stratified layer below the bubble.
- The gas distribution in the slugs is governed by forces on the bubbles.
- An inhomogeneous bubble distribution can give rise to a *distribution slip* (gas and liquid are distributed differently in regions of low and high velocity).
- This affects the average gas fraction in the slug.
The gas distribution in slugs (II)

- The gas is strongly skewed towards the top of the pipe, Nydal (1991)
The flow in the bubble zone

- The momentum equations for fully developed flow in the bubble zone read:

\[
0 = -\alpha_B \frac{\partial p}{\partial z} - \frac{\tau_{GB} S_{GB}}{A} - \frac{\tau_{IB} S_{IB}}{A} + G_{GB}
\]

\[
0 = -(1 - \alpha_B) \frac{\partial p}{\partial z} - \frac{\tau_{LB} S_{LB}}{A} + \frac{\tau_{IB} S_{IB}}{A} + G_{LB}
\]

- By eliminating the pressure gradient, one nonlinear algebraic equation is obtained for the gas fraction:

\[
(1 - \alpha_B) \left( \frac{\tau_{GB} S_{GB}}{A} + \frac{\tau_{IB} S_{IB}}{A} - G_{GB} \right) - \alpha_B \left( \frac{\tau_{LB} S_{LB}}{A} - \frac{\tau_{IB} S_{IB}}{A} - G_{LB} \right) = 0
\]
The flow in the bubble zone (II)

- To solve the holdup equation, the velocities \( U_{GS} \) and \( U_{LS} \) have to be computed for a given \( \alpha_B \).
- The continuity equations differ from pure stratified flow.
- For stratified flow, \( U_G = U_{SG} / \alpha \); \( U_L = U_{SL} / (1 - \alpha) \).
- For the stratified flow in the bubble region,

\[
U_{GB} = U_B + \frac{\alpha_S}{\alpha_B} (U_{GS} - U_B)
\]

\[
U_{LB} = U_B + \frac{1 - \alpha_S}{1 - \alpha_B} (U_{LS} - U_B)
\]
Three-phases gas-oil-water slug flow

- Low velocities: Oil and water separated (stratified)
- High velocities: Oil and water dispersed (mixed)
- Moderate velocities: Oil and water mixed in slug and separated in bubble zone (because $U_{LS} > U_{LB}$)
- Mixing of oil and water can increase frictional pressure drop (increased mixture viscosity)
Summary

• Qualitative description of slug flow
• Horizontal and vertical slug flow
• Transition from stratified to slug flow
• Slug formation, growth and collapse
• The unit cell model for gas-liquid slug flow
• Gas-oil-water slug flow – effect of mixing