

# MEK4600/9600: 3DWaveLab

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## 1 3D-PTV and Tomographic PIV measurements of periodic waves

For this assignment, volumetric velocity measurements of surface waves will be conducted using two complementary techniques: three-dimensional particle tracking velocimetry (3D-PTV) with the Shake-The-Box (STB) algorithm, and tomographic PIV (Tomo-PIV). Both results will be compared with a suitable mathematical model, e.g. Airy and Stokes wave theory. The goal is to generate linear/weakly nonlinear surface waves, measure the full three-dimensional velocity field beneath the free surface, and critically assess the strengths and limitations of each measurement approach.

The setup consists of a multi-camera volumetric system with a minimum of four cameras arranged to image an overlapping measurement volume. A pulsed volume illumination source (e.g. a diverging laser or pulsed LED array) will be used to illuminate the full measurement volume. Neutrally buoyant tracer particles (50  $\mu\text{m}$  polyamide spheres) should be used as flow tracers. Both techniques share this hardware; they differ in seeding density, reconstruction algorithm, and the nature of the result (Lagrangian tracks vs. Eulerian velocity volumes). Processing should be performed using an open-source or licensed volumetric PIV/PTV code (e.g. DaVis, OpenPTV, or MART-based tools).

### 3D-PTV with Shake-The-Box

The Shake-The-Box algorithm iteratively predicts particle positions from Lagrangian tracks established in earlier time steps, then “shakes” the reconstructed particles along their camera lines of sight to minimise residuals in each recorded image. This allows reliable tracking at source densities of  $N_s \approx 0.05\text{--}0.1$  particles per pixel (ppp). The output is a set of individual particle trajectories from which Lagrangian accelerations and Eulerian velocity fields can be derived.

### Tomographic PIV

Tomographic PIV reconstructs the three-dimensional light-intensity distribution of the seeded flow volume from the simultaneous images of the multiple cameras using a tomographic algorithm such as MART (Multiplicative Algebraic Reconstruction Technique). Cross-correlation of successive reconstructed intensity volumes yields a time-resolved Eulerian velocity field on a regular grid. Tomo-PIV operates at higher seeding densities ( $N_s \approx 0.05\text{--}0.15$  ppp) compared with classical PTV, but the spatial resolution is limited by the interrogation volume size, analogous to 2D PIV.

Please follow these steps:

1. Generate regular surface waves with frequency 2 Hz and focusing waves (breaking and non-breaking events).
2. Regular waves: find two appropriate amplitudes that generate a linear and a weakly nonlinear wave ( $ak < 0.3$ ). Remember to verify both  $kh$  and  $ak$ . Wave elevation must be measured with the resistance wire probes and compared to theory. Focusing waves: input data for generation will be provided.
3. Once the amplitudes and frequency are established, record volumetric image sequences for all wave conditions. Make at least three independent repetitions per condition. Note that Tomo-PIV and STB may require different seeding densities; adjust the particle concentration between runs if necessary or use the same seeding.
4. Perform camera calibration using a three-dimensional calibration target. Apply a volume self-calibration (VSC) step to reduce triangulation errors below 0.1 pixel before running either reconstruction algorithm.
5. **STB processing:** Run the STB algorithm and verify convergence by checking the residual image intensity after each “shake” iteration. Report the achieved tracking rate (percentage of particles successfully tracked between frames).
6. **Tomo-PIV processing:** Reconstruct the intensity volumes using the MART algorithm (or equivalent). Perform 3D cross-correlation with appropriate interrogation volume overlap ( $\geq 50\%$ ) and apply universal outlier detection to remove spurious vectors.
7. Extract the three-dimensional velocity field from both methods. Compare the measured velocity profiles below the wave crest with Airy and Stokes’ wave theory predictions. Airy:

$$u = a\omega \frac{\cosh k(z+h)}{\sinh kh} \cos(kx - \omega t), \quad (1)$$

$$w = a\omega \frac{\sinh k(z+h)}{\sinh kh} \sin(kx - \omega t). \quad (2)$$

Discuss any observed three-dimensional effects, e.g. transverse velocity components  $v$ .

8. Is it possible to detect Stokes drift from STB trajectories?
9. Dropouts in particle tracks (STB) and ghost particles in the reconstructed volume (Tomo-PIV) can both occur near the free surface; explain when and why they arise and describe how you handled them (e.g. masking the free-surface region in each camera image before reconstruction).
10. Comment on accuracy and difficulties in the measurements for each technique, including sensitivity to seeding density, camera synchronisation, and volume illumination uniformity.

## 2 Comparison of 3D-PTV (STB) and Tomographic PIV

Although both techniques use the same multi-camera hardware and volume illumination, they differ fundamentally in processing philosophy and output. The following points should be addressed and discussed:

1. **Nature of the result.** Tomo-PIV provides an Eulerian velocity field on a regular grid, directly analogous to standard 2D PIV. STB provides Lagrangian particle trajectories, from which both velocities and material accelerations can be obtained without additional differentiation in time.
2. **Seeding density.** Tomo-PIV requires a moderately dense seeding to ensure sufficient intensity in the reconstructed volume. STB can work at lower seeding densities because it relies on individual particle identification, but also extends reliably to higher densities by using track predictions to guide the reconstruction. Measure the actual particle image density ( $N_s$ ) achieved in your experiments and relate it to the expected performance of each method.
3. **Spatial resolution and dynamic range.** The spatial resolution of Tomo-PIV is set by the interrogation volume size, whereas STB resolves individual particle displacements. Compare the spatial resolution and dynamic velocity range achieved by the two methods in your measurements.
4. **Computational cost.** MART-based tomographic reconstruction is computationally expensive and scales poorly with volume size. STB is more efficient once the initial particle field is established, because it only tracks changes between frames. Comment on the practical processing times observed for each method.
5. **Accuracy near the free surface.** Both methods are challenged by the moving free surface, which causes changing illumination and masking complexity. Compare how each method handles the near-surface region and what masking strategies were required.
6. **Overall assessment.** Summarise when you would prefer Tomo-PIV over STB, and vice versa, for measurements of surface wave kinematics and for general fluid mechanics experiments.