# Lecture 2

## Proposition 3

### Proof

Only the case in which $m$ is changing.

To show that $V(t)$ increases when $m$ (compounding frequency) increase, it suffices to show

$$\left(1+\frac{r}{m}\right)^m < \left(1+\frac{r}{K}\right)^K, \quad m < K$$

$$\left(1+\frac{r}{m}\right)^m = \sum_{j=0}^{m} \frac{m!}{j!(m-j)!}\left(\frac{r}{m}\right)^j$$

$$= 1 + r + \sum_{j=2}^{m} \frac{r^j}{j!} \underbrace{\frac{m(m-1)(m-2)\cdots(m-j+1)}{m\times m\times\cdots\times m}}_{j \text{ times}}$$

$$= 1 + r + \sum_{j=2}^{m} \frac{r^j}{j!}\left(1-\frac{1}{m}\right)\left(1-\frac{2}{m}\right)\cdots\left(1-\frac{j-1}{m}\right)$$

$$< 1 + r + \sum_{j=2}^{m} \frac{r^j}{j!}\left(1-\frac{1}{K}\right)\left(1-\frac{2}{K}\right)\cdots\left(1-\frac{j-1}{K}\right)$$

$$< 1 + r + \sum_{j=2}^{K} \frac{r^j}{j!}\left(1-\frac{1}{K}\right)\left(1-\frac{2}{K}\right)\cdots\left(1-\frac{j-1}{K}\right)$$

$$= 1 + r + \sum_{j=2}^{K} \frac{r^j}{j!}\frac{K(K-1)\cdots(K-j+1)}{K\times\cdots\times K}$$

$$= \sum_{j=0}^{K} \frac{K!}{(K-j)!\,j!}\left(\frac{r}{K}\right)^j = \left(1+\frac{r}{K}\right)^K$$