Proof of Theorem 5.5

$\Rightarrow$) Suppose every contingent claim is attainable and suppose that there exist $Q, \hat{Q} \in M$, such that $Q \neq \hat{Q}$, i.e. $\exists\, k \in \{1, \dots, K\}$ such that $Q_k \neq \hat{Q}_k$.

Consider the claim

$$X(\omega) = \begin{cases} B(1) & \omega = \omega_k \\ 0 & \omega \neq \omega_k \end{cases}$$

Then,

$$E_Q\left[X/_{B(1)}\right] = Q_k \neq \hat{Q}_k = E_{\hat{Q}}\left[X/_{B(1)}\right]$$

which contradicts Lemma 4.4.

($\Leftarrow$) Let X be an arbitrary cont. claim and assume that $M = \{Q\}$. Then, trivially, $E_Q[X/B(1)]$ is constant over $Q \in M$. ($\exists! \, Q \in M$) $\Rightarrow$ the result follows again from Lemma 4.4.