# STK-MAT3700/4700
# Introduction to Mathematical Finance
# EXERCISE SET 9

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**Schedule note:** This set is presented in Week 44, Tuesday 27 October 2026, during the combined lecture and exercise meeting. The first hour is Lecture 13 and the second hour is Exercise Set 9. It corresponds to Lecture 12.

Main topics: two-period European option pricing, recombining binomial trees, risk-neutral expectations, backward induction, dividend-adjusted risk-neutral probabilities, CRR factors, and CRR pricing sums. Main book sections: Petters–Dong, Secs. 8.5.2–8.5.3 and 5.2.2; Pliska, Sec. 3.5 for the backward-induction viewpoint.

## Exercise 1. Two-Period European Call

**Source basis:** Based on Petters–Dong, Sec. 8.5.3, pp. 417–420. The numbers have been changed from the lecture example. The structure is also in the style of older two-step exam questions, for example the 2013 and 2024 exam problems on binomial trees.

Consider a two-period recombining binomial model with

$$S_0 = 100, \qquad u = 1.20, \qquad d = 0.90,$$

and one-period risk-free growth factor

$$e^{rh} = 1.05.$$

There are no dividends. Let $C$ be a European call option with strike $K = 105$ and maturity after two periods.

a) Compute the risk-neutral up-probability $p^*$.
b) Compute the possible stock prices at maturity and the corresponding call payoffs.
c) Price the call directly from the two-period risk-neutral expectation.
d) Price the same call by backward induction.
e) Explain why the middle payoff has coefficient $2p^*(1-p^*)$ in the direct formula.

## Exercise 2. Two-Period European Put

**Source basis:** Variation of Exercise 1, based on Petters–Dong, Sec. 8.5.3, pp. 417–420. This is included to make students practise the same tree with a different payoff.

Use the same two-period model as in Exercise 1:

$$S_0 = 100, \qquad u = 1.20, \qquad d = 0.90, \qquad e^{rh} = 1.05.$$

There are no dividends. Let $P$ be a European put option with strike $K = 105$ and maturity after two periods.

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*Date*: Autumn 2026.

a) Compute the terminal put payoffs.
b) Price the put directly from the two-period risk-neutral expectation.
c) Price the same put by backward induction.
d) Compare the call and put prices from Exercises 1 and 2. Explain why the put is positive even though the stock can go up.

## Exercise 3. Dividend Yield and the One-Step Probability

**Source basis:** Based on Petters–Dong, Sec. 8.5.2, pp. 415–417, and Petters–Dong, Sec. 5.2.2, pp. 230–236. This exercise isolates the dividend-adjusted risk-neutral probability before larger CRR trees.

Consider a one-period binomial model with time step

$$h = \frac{1}{12}.$$

The stock starts at $S_0 = 50$ and moves to $S_0u$ or $S_0d$, where

$$u = 1.15, \qquad d = 0.92.$$

The continuously compounded risk-free rate is $r = 0.06$, and the continuous dividend yield is $\delta = 0.02$.

Let $X$ be a one-period derivative with terminal payoff

$$X^u = 6 \qquad \text{after an up move,}$$

and

$$X^d = 1 \qquad \text{after a down move.}$$

a) Check the one-step no-arbitrage inequality.
b) Compute the risk-neutral probability $p^*$ and $1 - p^*$.
c) Compute the no-arbitrage price of $X$ at time 0.
d) Explain why the dividend yield affects $p^*$, but the derivative payoff is discounted with $e^{-rh}$.

## Exercise 4. A Four-Step CRR Call

**Source basis:** Based on Petters–Dong, Sec. 5.2.2, pp. 230–236, and the CRR questions prepared in the exam-problem bank. This is a shorter computational variant, not a copy of an old exam.

Consider a CRR model with

$$S_0 = 100, \qquad K = 100, \qquad T = 1,$$

and $n = 4$ time steps. The continuously compounded risk-free rate is $r = 0.04$, the continuous dividend yield is $\delta = 0.01$, and the volatility parameter is $\sigma = 0.20$.

Use

$$h_n = \frac{T}{n}, \qquad u_n = e^{\sigma\sqrt{h_n}}, \qquad d_n = e^{-\sigma\sqrt{h_n}}.$$

a) Compute $h_n$, $u_n$, and $d_n$.
b) Compute the CRR risk-neutral probability $p_n^*$.
c) Check the no-arbitrage inequality.
d) Compute the terminal stock prices for $k = 0, 1, 2, 3, 4$ up moves.
e) Compute the European call price using the CRR binomial sum.