Eksempler på tettheter

Uniform fordeling

La X være en kontinuerlig stok. var.
som kan ta verdier i $[a, b]$.

Anta at X er uniformt fordelt. Det betyr at tetthetsen er konstant mellom a og b

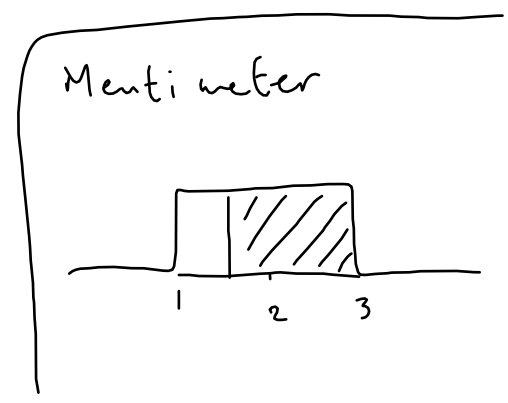
Må ha at $f(x) = c$ for alle $x \in [a, b]$

I tillegg må vi ha

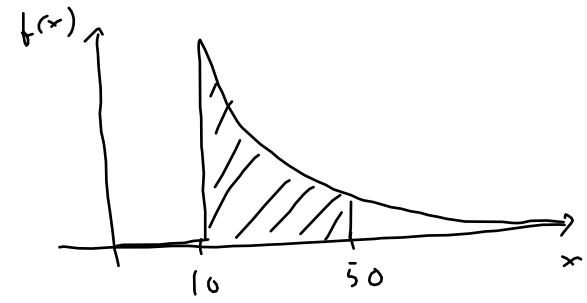
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Det gir

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{hvis } x \in [a, b] \\ 0 & \text{ellers} \end{cases}$$



Eksempel 4.3 boken



X : skadelig flomhastighet (m^3/s)

$$f(x) = \begin{cases} 0.04\, e^{-0.04(x-10)} & x \geq 10 \\ 0 & \text{ellers} \end{cases}$$

Er dette en tetthet? Er $f(x) \geq 0 \ \forall x$ JA

Er $\int_{-\infty}^{\infty} f(x)\,dx = 1$? må regne!

substituerer $t = x - 10$

$$\int_{-\infty}^{\infty} f(x)\,dx = \underbrace{\int_{-\infty}^{10} 0\,dx}_{=0} + \int_{10}^{\infty} 0.04\, e^{-0.04(x-10)}\,dx$$

$$= \int_{0}^{\infty} 0.04\, e^{-0.04t}\,dt = -e^{-0.04t}\Big]_0^{\infty} = 0 - (-1) = 1 \quad \text{JA!}$$

flva er sannsynligheten for at flomhastigheten er maks 50 m^3/s ?

$$P(X \leq 50) = \int_{-\infty}^{50} f(x)\,dx = \int_{10}^{50} 0.04\, e^{-0.04(x-10)}\,dx$$

substit. $t = x - 10$

$$= \int_{0}^{40} 0.04\, e^{-0.04t}\,dt = -e^{-0.04t}\Big]_0^{40}$$

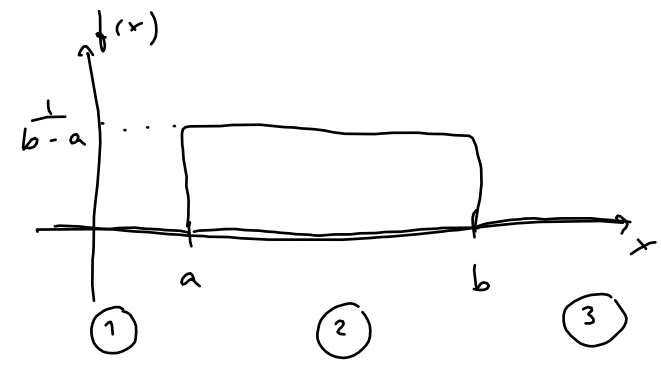
$$= -e^{-0.04 \cdot 40} - (-e^{-0}) = 1 - e^{-1.6} \approx 0.798$$

Kumulativ fordelingsfunksjon

Uniform fordeling

$X \sim \text{uniform}(a, b)$

$$f(x) = \begin{cases} \frac{1}{b-a} & x \in (a, b) \\ 0 & \text{ellers} \end{cases}$$



Vil finne $F(x)$: Deler opp i områder

① $x \leq a$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-\infty}^{x} 0\,dt = 0$$

② $x \in (a, b)$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt = \underbrace{\int_{-\infty}^{a} 0\,dt}_{=0} + \int_{a}^{x} \frac{1}{b-a}\,dt$$

$$= \frac{t}{b-a}\Big]_{a}^{x} = \frac{x-a}{b-a}$$

③ $x \geq b$

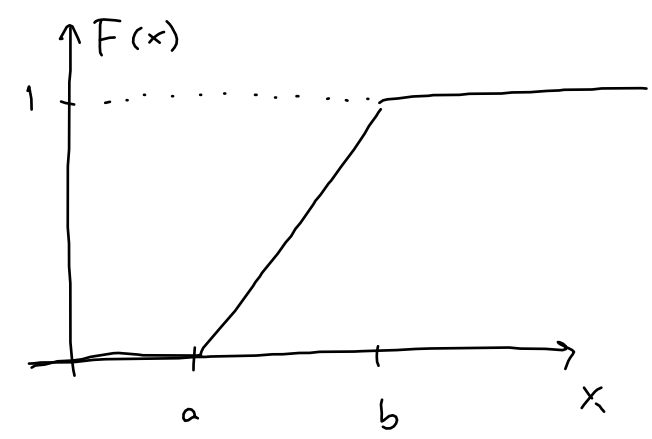
$$F(x) = P(X \leq x) = \cancel{\int_{-\infty}^{a} 0\,dt} + \int_{a}^{b} \frac{1}{b-a}\,dt + \cancel{\int_{b}^{x} 0\,dt}$$

$$= \frac{t}{b-a}\Big]_{a}^{b} = \frac{b-a}{b-a} = 1$$

Så

$$F(x) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & x \in (a, b) \\ 1 & x \geq b \end{cases}$$

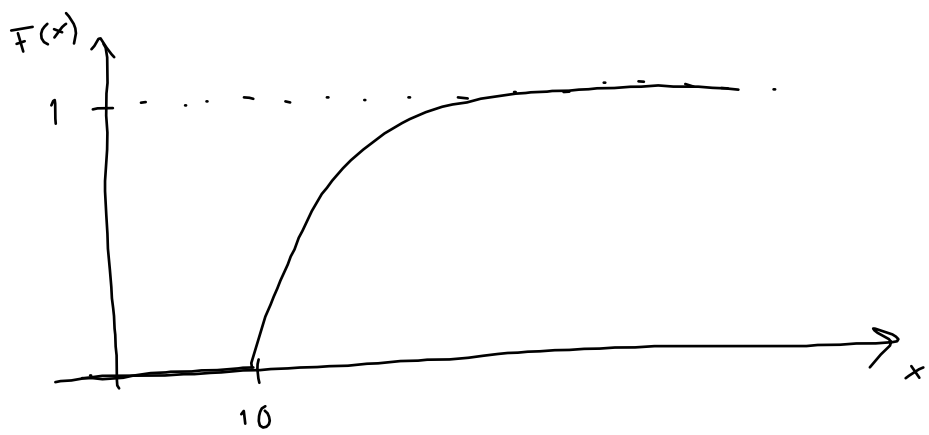
Uniform fordeling!



Strømningshastighet

Overbevis deg selv (hjemmelekse!) om at

$$F(x) = \begin{cases} 0 & x \leq 10 \\ 1 - e^{-0.04(x-10)} & x > 10 \end{cases}$$



Persentiler Flomhastighet

$$F(x) = \begin{cases} 0 & x \leq 10 \\ 1 - e^{-0.04(x-10)} & x > 10 \end{cases}$$

La $p \in [0,1]$ og $\eta(p)$ tilhørende persentil

$$p = F(\eta(p)) \qquad x > 10$$

$$p = 1 - e^{-0.04(\eta(p)-10)}$$ løser denne mhp. $\eta(p)$

$$1 - p = e^{-0.04(\eta(p)-10)}$$

$$\ln(1-p) = -0.04(\eta(p)-10)$$

$$\eta(p) = 10 - \frac{1}{0.04}\ln(1-p)$$

Eks. median

$$\tilde{\mu} = \eta(0.5) = 10 - \frac{1}{0.04}\ln(1-0.5) = \underline{27.33\ m^3/s}$$

Menti

