Forventningsverdi $\mu_X = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

diskret: $\sum_x x\, p(x)$

Uniform fordeling

$X \sim \text{Uniform}(a, b)$ $\qquad f(x) = \begin{cases} \frac{1}{b-a} & x \in (a, b) \\ 0 & \text{ellers} \end{cases}$

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_a^b x \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a} \cdot \frac{1}{2} x^2 \Big]_a^b = \frac{1}{2(b-a)} (b^2 - a^2) = \frac{1}{2(b-a)} (b-a)(b+a)$$

$$= \frac{b+a}{2}$$

Forventet flomhastighet



$$X \sim f(x) = \begin{cases} 0 & x \le 10 \\ 0.04 e^{-0.04(x-10)} & x > 10 \end{cases}$$

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{10}^{\infty} x \cdot 0.04 e^{-0.04(x-10)}\,dx$$

subst: $t = x - 10$

$$= \int_{-\infty}^{\infty} (t+10)\, 0.04 e^{-0.04t}\,dt$$

$$= 10 \underbrace{\int_0^{\infty} 0.04 e^{-0.04t}\,dt}_{=1} + \int_0^{\infty} t \cdot 0.04 e^{-0.04t}\,dt$$

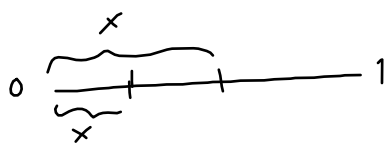
SJEKK! må bruke delvis integrasjon

$u \quad dv$

$$= 10 + \int_0^{\infty} e^{-0.04t}\,dt = 10 + \left(-\frac{1}{0.04} e^{-0.04t} \right]_0^{\infty}$$

$$= 10 + \frac{1}{0.04} = 10 + 25 = \underline{35}$$

Eksempel "broken stick" $\boxed{E(h(X))}$



1 enhet lang

knekker på tilfeldig sted

X : lengden fram til knekkpunktet fra venstre

$X \sim$ Uniform $(0,1)$

$$f(x) = \begin{cases} 1 & x \in (0,1) \\ 0 & \text{ellers} \end{cases}$$

Y : lengden av den lengste biten

Vil finne $E(Y)$ - her blir $Y = h(X)$

$$Y = \begin{cases} 1-X & x \leq 1/2 \\ X & x > 1/2 \end{cases}$$

$$E(Y) = E(h(X)) = \int_{-\infty}^{\infty} h(x) f(x)\, dx$$

$$= \int_{0}^{1/2} (1-x) f(x)\, dx + \int_{1/2}^{1} x f(x)\, dx$$

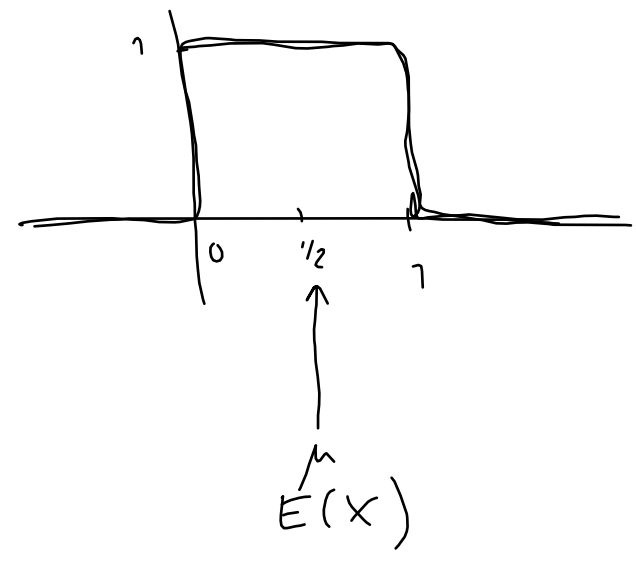
$$= \int_{0}^{1/2} (1-x)\, dx + \int_{1/2}^{1} x\, dx$$

$$= \left[x - \frac{1}{2}x^2 \right]_0^{1/2} + \left[\frac{1}{2}x^2 \right]_{1/2}^{1}$$

$$= \frac{1}{2} - \frac{1}{8} + \frac{1}{2} - \frac{1}{8} = \frac{3}{4}$$

Varians uniform fordeling

$X \sim$ Uniform $(0,1)$



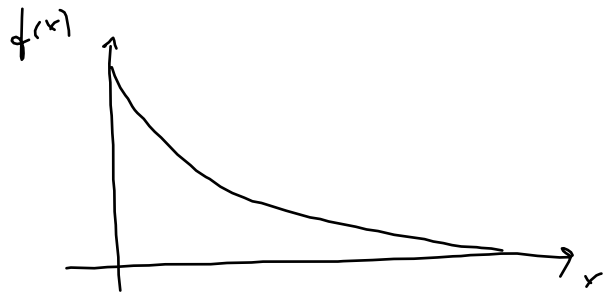
$$E(X) = \int_0^1 x\,dx = \frac{1}{2}x^2\Big]_0^1 = \underline{\underline{1/2}}$$

$$E(X^2) = \int_0^1 x^2\,dx = \frac{1}{3}x^3\Big]_0^1 = 1/3$$

$$V(X) = E(X^2) - \left[E(X)\right]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \underline{\underline{\frac{1}{12}}}$$

MGF kontinuerlig tilfelle





eksponensialfordeling

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \quad , \quad \lambda > 0 \\ 0 & \text{ellers} \end{cases}$$

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx = \int_0^{\infty} e^{tx} \lambda e^{-\lambda x}\,dx$$

$$= \lambda \int_0^{\infty} e^{-(\lambda - t)x}\,dx$$

$$= \lambda \left[-\frac{1}{(\lambda - t)} e^{-(\lambda - t)x} \right]_0^{\infty}$$

$$= \frac{\lambda}{\lambda - t}$$

MGF eksponensialfordelingen

Deriverer $M_X(t)$ (mhp t)

$$M_X'(t) = \frac{\lambda}{(\lambda - t)^2}$$

SJEKK DETTE SELV!

$$M_X''(t) = \frac{2\lambda}{(\lambda - t)^3}$$

$$E(X) = M_X'(0) = \frac{\lambda}{(\lambda - 0)^2} = \frac{1}{\lambda}$$

$$E(X^2) = M_X''(0) = \frac{2\lambda}{(\lambda - 0)^3} = \frac{2}{\lambda^2}$$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$