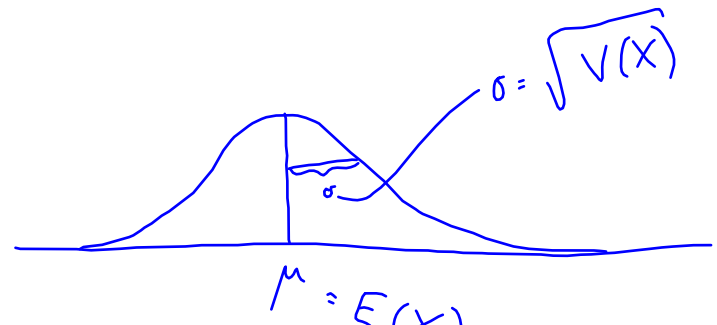
Recap Normalfordelingen kap 4.3

$$X \sim N(\mu, \sigma^2)$$



$$\frac{X-\mu}{\sigma} = Z \sim N(0,1)$$ ← standard normalfordeling

$$P(Z \le z) = F(z) = \Phi(z)$$ kumulativ fordeling for $N(0,1)$

↖ kan slår opp i tabell

Momentgen. funk. for normalfordeling

$X \sim N(\mu, \sigma^2)$ $\quad Z = \frac{X-\mu}{\sigma} \sim N(0,1)$ $\quad X = \sigma Z + \mu$ ⊛

Finner først MGF til Z:

$$M_Z(t) = E\left(e^{tZ}\right) = \int_{-\infty}^{\infty} e^{tz} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z^2 - 2tz)}\, dz$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z^2 - 2tz + t^2) + t^2/2}\, dz$$ triks!

$$= e^{t^2/2} \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z-t)^2}\, dz}_{N(t,1)} = e^{t^2/2}$$

$= 1$

Ser så på X

$$M_X(t) = E\left(e^{tX}\right) \overset{⊛}{=} E\left(e^{t(\sigma Z + \mu)}\right) = E\left(e^{t\sigma Z} \cdot e^{t\mu}\right)$$

$$= e^{t\mu} E\left(e^{(t\sigma)Z}\right) = e^{t\mu} M_Z(t\sigma)$$

$$= e^{t\mu} e^{t^2\sigma^2/2} = \underline{\underline{e^{t\mu + \sigma^2 t^2/2}}}$$

4.4 Gammafordeling

X stok. var tetthet på formen

$$f(x) = \begin{cases} K \cdot x^{\alpha-1} e^{-x/\beta} & x > 0 \\ 0 & \text{ellers} \end{cases}$$

Hva må K være? Vi må ha $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} K x^{\alpha-1} e^{-x/\beta}\,dx = K \int_0^{\infty} x^{\alpha-1} e^{-x/\beta}\,dx$$

substitusjon:
$u = x/\beta$
$x = \beta u$
$dx = \beta\,du$

$$= K \int_0^{\infty} (\beta u)^{\alpha-1} e^{-u} \beta\,du$$

$$= K\beta^{\alpha} \underbrace{\int_0^{\infty} u^{\alpha-1} e^{-u}\,du}_{\Gamma(\alpha)}$$

gammafunksjonen

$$= K\beta^{\alpha}\Gamma(\alpha)$$

For at integral = 1 må $K = \frac{1}{\beta^{\alpha}\Gamma(\alpha)}$

Tetthetsen blir derfor

$$f(x) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} & x > 0 \\ 0 & \text{ellers} \end{cases}$$

Gammafunktionen

$$\Gamma(\alpha) = \int_0^\infty u^{\alpha-1} e^{-u}\, du \qquad \alpha > 0$$

1) $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1) \qquad \alpha > 1$

2) $\Gamma(n) = (n-1)!$

n positivt heltall

3) $\Gamma(1/2) = \sqrt{\pi}$

Viser 1)

$$\underline{\Gamma(\alpha)} = \int_0^\infty \underbrace{u^{\alpha-1}}\ \underbrace{e^{-u}}\, du = \left[u^{\alpha-1}(-e^{-u}) \right]_0^\infty - \int_0^\infty (\alpha-1) u^{\alpha-2}(-e^{-u})\, du$$

delvis integrasjon

$$= 0 + (\alpha-1)\underbrace{\int_0^\infty u^{\alpha-2} e^{-u}\, du}_{\Gamma(\alpha-1)} = \underline{\underline{(\alpha-1)\Gamma(\alpha-1)}}$$