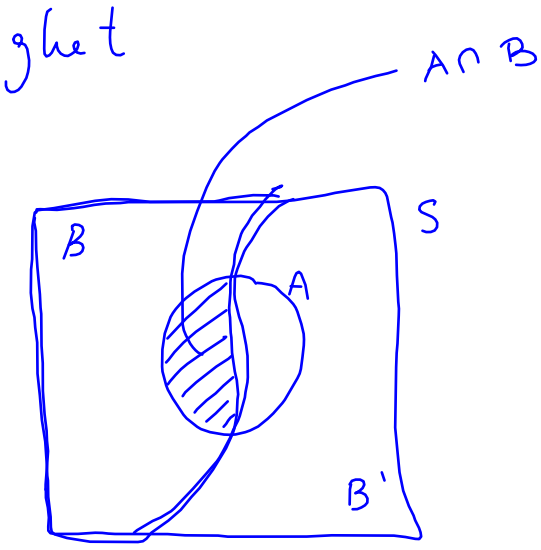
Rep. 2.4 Betinget sannsynlighet

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

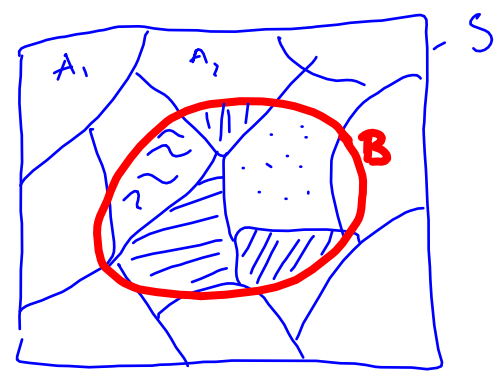


Produktsetningen

$$P(A \cap B) = P(A \mid B) \cdot P(B)$$

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2 \mid A_1) \cdot P(A_3 \mid A_1 \cap A_2)$$

Setningen om total sannsynlighet



Hvis $A_1 \dots A_k$ er disjunkte

og $A_1 \cup A_2 \dots \cup A_k = S$

$$P(B) = \sum_{j=1}^{k} P(A_j \cap B)$$

produktsetningen

$$= \sum_{j=1}^{k} P(B \mid A_j) \cdot P(A_j)$$

brukes i situasjoner der $P(B \mid A_j)$ er enklere å finne

Forrige gang: også Bayes setning

Slide 25 : Hvis $\overbrace{P(A \mid B) = P(A)}^{(*)}$ så er $P(B \mid A) = P(B)$

skal vises

Bevis:

$$P(B \mid A) \overset{\text{def.}}{=} \frac{P(A \cap B)}{P(A)} \overset{\text{produkt-setn.}}{=} \frac{P(A \mid B) \cdot P(B)}{P(A)}$$

$$\overset{(*)}{=} \frac{\cancel{P(A)} \cdot P(B)}{\cancel{P(A)}} = P(B)$$

Slide 27

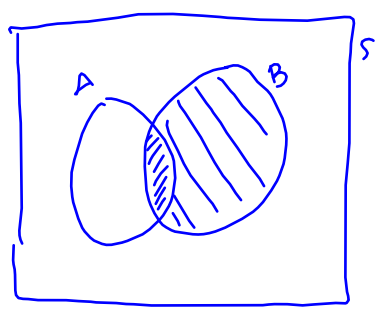
Viser den første

Anta $P(A \mid B) = P(A)$ dvs. A og B er uavh.

Vil vise at da er også A' og B uavh. dvs. $P(A' \mid B) = P(A')$

Har at

$$P(A' \mid B) = \frac{P(A' \cap B)}{P(B)} \quad (**)$$



$$B = (A \cap B) \cup (A' \cap B)$$

$$P(B) = P(A \cap B) + P(A' \cap B) \quad \text{(disjunkte)}$$

dvs. $P(A' \cap B) = P(B) - P(A \cap B)$

Setter inn i $(**)$

Får da

$$\underline{\underline{P(A' \mid B)}} = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$$

$$= 1 - \frac{P(A \cap B)}{P(B)} = 1 - P(A \mid B)$$

$$= 1 - P(A) = \underline{\underline{P(A')}}$$

Også slide 27:

Viktig resultat

$$P(A \cap B) = P(A) \cdot P(B)$$
hvis og bare hvis
A og B er uavhengige

Bevis: Skal vise $P(A|B) = P(A)$

$$\Updownarrow$$

$$P(A \cap B) = P(A) \cdot P(B)$$

Anta først $P(A|B) = P(A)$. Da finner vi

$$\underline{P(A \cap B)} \overset{\text{prod. setn.}}{=} P(A|B) \cdot P(B) = \underline{P(A) \cdot P(B)}$$

Anta så $P(A \cap B) = P(A) \cdot P(B)$. Da finner vi

$$\underline{P(A|B)} \overset{\text{def.}}{=} \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = \underline{P(A)}$$

Slide 29

Uavhengighet for tre hendelser:

A, B og C er uavhengige dersom følgende gjelder:

$$\left.\begin{aligned} P(A \cap B) &= P(A) \cdot P(B) \\ P(A \cap C) &= P(A) \cdot P(C) \\ P(B \cap C) &= P(B) \cdot P(C) \end{aligned}\right\} \text{ i eksempelet!}$$

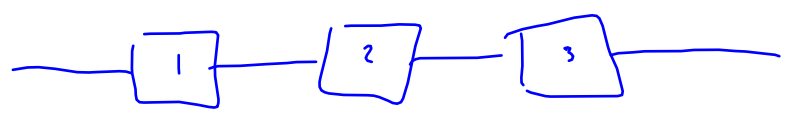
$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

Dette impliserer f.eks.

$$P(A \mid B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{P(A) \cdot P(B) \cdot P(C)}{P(B) \cdot P(C)} = P(A)$$

Eksempel 16

Seriesystem



A_i = komponent i fungerer

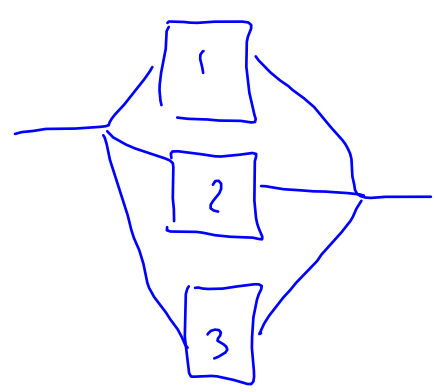
$P(A_i) = 0.9 \quad \forall i$

Antar uavhengighet

$$P(\text{system fungerer}) = P(A_1 \cap A_2 \cap A_3) \overset{\text{uavh.}}{=} P(A_1) \cdot P(A_2) \cdot P(A_3)$$

$$= 0.9^3 = 0.729$$

Parallell system



$$P(\text{system fungerer}) = P(\text{minst en komponent fungerer})$$

$$= 1 - P(\text{ingen fungerer})$$

$$= 1 - P(A_1' \cap A_2' \cap A_3')$$

$$\overset{\text{uavh.}}{=} 1 - P(A_1') \cdot P(A_2') \cdot P(A_3')$$

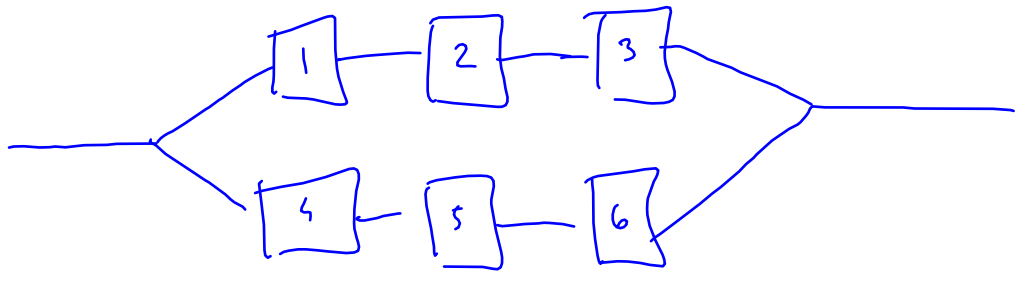
$$= 1 - (0.1)^3 = 0.999$$

Slide 35 ekv. 2.37 i boken

A_i = komponent i fungerer

$P(A_i) = 0.9$

Antar alle oavhengige



System fungerer : $(A_1 \cap A_2 \cap A_3) \cup (A_4 \cap A_5 \cap A_6)$

$P(\text{system fungerer}) = P(A_1 \cap A_2 \cap A_3) + P(A_4 \cap A_5 \cap A_6) - P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5 \cap A_6)$

oavh. $= 0.9^3 + 0.9^3 - 0.9^6 = 0.927$

Siste

