Plenum STK 1110

Uke 43

31 $x = CO_2$ - konsentrasjon

y = drømasse

$\beta_1 = 0.008$

$\sigma = 0.5$

$Y = \beta_0 + \beta_1 x \cdot \varepsilon$

a) $\sigma_{\hat{\beta}_1} = \dfrac{\sigma}{\sqrt{S_{xx}}}$

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n} = 70\,000$$

$$\sigma_{\hat{\beta}_1} = \frac{0.5}{\sqrt{70000}} = \underline{\underline{0.00189}}$$

b) $P(0.006 \leq \hat{\beta}_1 \leq 0.010)$

$$= P\left(\frac{0.006 - \overset{E[\hat{\beta}_1]}{\overset{\shortparallel}{\beta_1}}}{\sigma_{\hat{\beta}_1}} \leq Z \leq \frac{0.010 - \beta_1}{\sigma_{\hat{\beta}_1}}\right)$$

$$= P\left(\frac{0.006 - 0.008}{0.00189} \leq Z \leq \frac{0.010 - 0.008}{0.00189}\right)$$

$$= P(-1.058 \leq Z \leq 1.058)$$

$$= \Phi(1.058) - \Phi(-1.058)$$

$$= 0.855 - 0{,}145$$

$$= \underline{\underline{0{,}71}}$$

c) Finner $S_{xx} = 24\,750 < 70000$

Dette vil gjøre $\sigma_{\hat{\beta}_1}$ større.

Vi foretrekker der for flere obs. som er spredt i dette tilfellet.

(32) x = nedbørsmengde

y = hvor mye vann som renner av motorveien

a) Er det en lineær sammenheng mellom x og y?

$H_0 : \beta_1 = 0$ mot $H_a : \beta_1 \neq 0$.

$$T = \frac{\hat{\beta}_1 - \beta_1}{S_{\hat{\beta}_1}} \sim t_{n-2}$$

$$t = \frac{0.82697 - 0}{0.03652} = 22,65$$

p-verdi : 0.000

Vi forkaster H_0, så det er en lineær sammenheng.

K.i. for gj. snittlig endring i y når x øker med 1.

$n = 15$

$\hat{\beta}_1 \pm t_{\alpha/2, n-2} \cdot s_{\hat{\beta}_1}$

$\alpha = 0.05$

$\hat{\beta}_1 \pm t_{0.025, 13} \cdot s_{\hat{\beta}_1}$

$= 0.82697 \pm 2,16 \cdot 0,03652$

$= \underline{\underline{(0.748 , 0.906)}}$

(34)

$y = \beta_0 + \beta_1 x + \varepsilon$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \underline{\underline{9.96}}$$

$$\hat{\beta}_0 = \frac{\sum y_i - \hat{\beta}_1 \sum x_i}{n} = \bar{y} - \hat{\beta}_1 \bar{x} = \underline{\underline{-305,9}}$$

Linear regresjonsmodell:

$y = -305,9 + 9,96\,x$

$SSE = \sum y_i^2 - \hat{\beta}_0 \sum y_i - \hat{\beta}_1 \sum x_i y_i = 2\,900\,807$

$SST = \sum y_i^2 - \frac{(\sum y_i)^2}{n} = 3\,310\,341$

$r^2 = 1 - \frac{SSE}{SST} = \underline{\underline{0,124}}$

Bare 12,4 % av variansen i y kan forklares v.h.a. regresjonsmodellen.

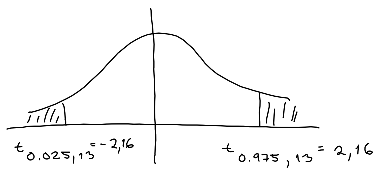
b) $H_0: \beta_1 = 0$, $H_a: \beta_1 \neq 0$

$$T = \frac{\hat{\beta}_1 - \beta_1}{s_{\hat{\beta}_1}} \sim t_{n-2} = t_{13}$$

$s^2 = \frac{SSE}{n-2} = 223\,139$

$s_{\hat{\beta}_1} = \frac{s}{\sqrt{S_{xx}}} = 7,35$

$t = \frac{9,96}{7,35} = 1,35$



$-2,16 < t < 2,16$, så vi forkaster ikke H_0.

c) Fjerner outlieren

$\hat{\beta}_1 = 0.78$

$\hat{\beta}_0 = 34,37$

$r^2 = 0,024$

For hypotesetesten finner vi:

$t = 0.54$

$t_{0.025,\,12} = -2,179$

$t_{0.975,\,12} = 2,179$

$-2,179 < t < 2,179$, så vi forkaster ikke H_0.

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$$\hat{y} = \underbrace{-1{,}128}_{\hat{\beta}_0} + \underbrace{0{,}82697}_{\hat{\beta}_1} x$$

$r^2 = 0{,}975$

$s = 5{,}24$

$n = 15$

a) $x^* = 40$

$\hat{y}^* = -1{,}128 + 0{,}82697 \cdot 40 = 31{,}95$

$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}} = 1{,}44$ (oppgitt i oppgaven)

95 % prediksjonsintervall for $\hat{y}^*$:

$\hat{\beta}_0 + \hat{\beta}_1 \cdot x^* \pm t_{0.025,\, 13} \cdot \sqrt{s_{\hat{y}^*}^2 + s^2}$

$= 31.95 \pm 2{,}16 \cdot \sqrt{1{,}44^2 + 5{,}24^2}$

$= \underline{\underline{(20.21 \;,\; 43.69)}}$

b) $x^* = 50$

$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}}$

$= 5{,}24 \sqrt{\frac{1}{15} + \frac{(50 - 53.2)^2}{20584{,}6}}$

$= 1{,}358$

95 % p.i. for $s_{\hat{y}^*}$:

$\hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{0.025,\, 13} \cdot \sqrt{s^2 + s_{\hat{y}^*}^2}$

$= \underline{\underline{(28{,}53 \;,\; 51{,}91)}}$

Det simultane prediksjonsnivået er :
Minst $100(1 - 2\alpha)\,\% = \underline{\underline{90\,\%}}$

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$$y = \underbrace{1008{,}14}_{\hat{\beta}_0} + \underbrace{6{,}19268}_{\hat{\beta}_1} \; x$$

$$s = 7{,}265$$

$$r^2 = 0{,}968$$

a) Test av modellnytten :

$H_0 : \beta_1 = 0$, $H_a : \beta_1 \neq 0$

$$T = \frac{\hat{\beta}_1 - 0}{s_{\hat{\beta}_1}} \sim t_{n-2}$$

$$s_{\hat{\beta}_1} = \frac{s}{\sqrt{s_{xx}}} = \frac{7{,}265}{\sqrt{165{,}63}} = 0{,}565$$

$$t = \frac{6{,}19268}{0{,}565} = 10.96$$

p-verdi : $P(T > t) = 1 - P(T \leq t)$

$= 1 - pt(10.96 \, , \, 4)$ (i R)

$= 0.0002$

Forkaster H_0 .

b) $e_i = y_i - \hat{y}_i$

c) $$e_i^* = \frac{y_i - \hat{y}_i}{s\sqrt{1 - \frac{1}{n} - \frac{(x_i - \bar{x})^2}{s_{xx}}}}$$