STK1110 - Statistiske metoder og dataanalyse høsten 2019
Løsningsforslag til ekstraoppgave for uke 43

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Ekstraoppgave: Vis formel (12.4) på side 636 i læreboka.

Vi vil vise at

$$\text{SST} = \text{SSE} + \text{SSR}$$

der $\text{SST} = \sum_{i=1}^{n} (y_i - \overline{y})^2$, $\text{SSE} = \sum_{i=1}^{n} (y_i - \widehat{y}_i)^2$, $\text{SSR} = \sum_{i=1}^{n} (\widehat{y}_i - \overline{y})^2$.

Vi kan skrive

$$\begin{aligned}
\text{SST} &= \sum_{i=1}^{n} (y_i - \overline{y})^2 = \sum_{i=1}^{n} (y_i - \widehat{y}_i + \widehat{y}_i + \overline{y})^2 \\
&= \sum_{i=1}^{n} \left[(y_i - \widehat{y}_i)^2 + (\widehat{y}_i - \overline{y})^2 + 2\,(y_i - \widehat{y}_i)\,(\widehat{y}_i - \overline{y}) \right] \\
&= \sum_{i=1}^{n} (y_i - \widehat{y}_i)^2 + \sum_{i=1}^{n} (\widehat{y}_i - \overline{y})^2 + 2 \sum_{i=1}^{n} (y_i - \widehat{y}_i)\,(\widehat{y}_i - \overline{y}) \\
&= \text{SSE} + \text{SSR} + 2 \sum_{i=1}^{n} (y_i - \widehat{y}_i)\,(\widehat{y}_i - \overline{y})\,.
\end{aligned}$$

Altså er vi ferdig med beviset hvis vi kan vise at $\sum_{i=1}^{n} (y_i - \widehat{y}_i)\,(\widehat{y}_i - \overline{y}) = 0$.

Ved å bruke at $\widehat{y}_i = \widehat{\beta}_0 + \widehat{\beta}_1 x_i$ og ligningene fra side 626 i læreboka, dvs.

$$\sum_{i=1}^{n} \left(y_i - \widehat{\beta}_1 x_i - \widehat{\beta}_0 \right) = 0 \quad \text{og} \quad \sum_{i=1}^{n} x_i \left(y_i - \widehat{\beta}_1 x_i - \widehat{\beta}_0 \right) = 0$$

, får vi

$$\begin{aligned}
\sum_{i=1}^{n} (y_i - \widehat{y}_i)\,(\widehat{y}_i - \overline{y}) &= \sum_{i=1}^{n} \left(y_i - \widehat{\beta}_0 - \widehat{\beta}_1 x_i \right) \left(\widehat{\beta}_0 + \widehat{\beta}_1 x_i - \overline{y} \right) \\
&= \widehat{\beta}_1 \sum_{i=1}^{n} x_i \left(y_i - \widehat{\beta}_1 x_i - \widehat{\beta}_0 \right) + \left(\widehat{\beta}_0 - \overline{y} \right) \sum_{i=1}^{n} \left(y_i - \widehat{\beta}_1 x_i - \widehat{\beta}_0 \right) \\
&= \widehat{\beta}_1 \cdot 0 + \left(\widehat{\beta}_0 - \overline{y} \right) \cdot 0 \\
&= 0
\end{aligned}$$

Dermed har vi bevist at $\text{SST} = \text{SSE} + \text{SSR}$.