

Fasit til ukeoppgaver med gjennomgang 10. mai

Eksamen i ST114 mai 2003, Oppgave 2

a) Vi har

$$M(n) = \left(\frac{1-p}{p}\right)^n$$

og

$$\lim_{n \rightarrow \infty} M(n) = \begin{cases} \infty & \text{if } p < \frac{1}{2} \\ 1 & \text{if } p = \frac{1}{2} \\ 0 & \text{if } p > \frac{1}{2} \end{cases}$$

b)

$$\lim_{n \rightarrow \infty} \text{Prob}(X_n = 0) = \frac{p}{1-p}, \quad p < \frac{1}{2}$$

$$\lim_{n \rightarrow \infty} \text{Prob}(X_n = 0) = 1, \quad p \geq \frac{1}{2}$$

d)

$$\begin{aligned} \phi_2(s) &= \frac{p(1-qs)}{1-qp-qs} \\ P(X_2 = 0) &= \phi_2(0) = \frac{p}{1-qp} \\ P(X_2 = k) &= p^2 \left(\frac{q}{1-pq}\right)^{k+1}, \quad k = 1, 2, \dots \end{aligned}$$

Eksamen i STK1130 juni 2005, Oppgave 3

a) $m_n = m^n$

b) $P(X_n = 0) = h_n(0)$

d) Vi har

$$m_n = m^n = \left(\frac{q}{p}\right)^n$$

og

$$\lim_{n \rightarrow \infty} m_n = \begin{cases} \infty & \text{if } p < \frac{1}{2} \\ 1 & \text{if } p = \frac{1}{2} \\ 0 & \text{if } p > \frac{1}{2} \end{cases}$$

e)

$$\lim_{n \rightarrow \infty} \text{Prob}(X_n = 0) = \frac{p}{1-p}, \quad p < \frac{1}{2}$$

$$\lim_{n \rightarrow \infty} \text{Prob}(X_n = 0) = 1, \quad p \geq \frac{1}{2}$$

f)

$$P(X_n = 0) = \begin{cases} \frac{m^n - 1}{m^{n+1} - 1} & \text{hvis } m \neq 1 \\ \frac{n}{n+1} & \text{hvis } m = 1 \end{cases}$$

g) For $p = q = \frac{1}{2}$:

$$P(T \leq n) = \frac{n}{n+1}$$

$$P(T = n) = \frac{1}{n(n+1)}, \quad n = 1, 2, \dots$$

Ekstraoppgave 9

c)

$$P(X_n = 0) = 0$$

$$P(X_n = j) = \frac{(1 - \frac{1}{2^n})^{j-1}}{2^n}, \quad j > 0$$

$$E[X_n] = 2^n$$