2. We will now derive the probability that a given observation is part of a bootstrap sample. Suppose that we obtain a bootstrap sample from a set of n observations.

   (a) What is the probability that the first bootstrap observation is *not* the jth observation from the original sample? Justify your answer.

   (b) What is the probability that the second bootstrap observation is *not* the jth observation from the original sample?

   (c) Argue that the probability that the jth observation is *not* in the bootstrap sample is $(1 - 1/n)^n$.

   (d) When $n = 5$, what is the probability that the jth observation is in the bootstrap sample?

   (e) When $n = 100$, what is the probability that the jth observation is in the bootstrap sample?

   (f) When $n = 10,000$, what is the probability that the jth observation is in the bootstrap sample?

   (g) Create a plot that displays, for each integer value of n from 1 to $100,000$, the probability that the jth observation is in the bootstrap sample. Comment on what you observe.

   (h) We will now investigate numerically the probability that a bootstrap sample of size $n = 100$ contains the jth observation. Here $j = 4$. We repeatedly create bootstrap samples, and each time we record whether or not the fourth observation is contained in the bootstrap sample.

```
> store=rep(NA, 10000)
> for(i in 1:10000){
    store[i]=sum(sample(1:100, rep=TRUE)==4)>0
}
> mean(store)
```

Comment on the results obtained.