Week 13

6a) $\mathrm{EPE}(f) = \int_x \int_y L(y, f(x))\, p(x,y)\, dy\, dx$

$= \int_x \int_y L(y, f(x))\, p(y|x) p(x)\, dy\, dx$

$= \int_x \left[\int_y L(y, f(x))\, p(y|x)\, dy\right] p(x)\, dx$

$= \int_x \underbrace{\left[\sum_{y=0}^{1} L(y, f(x))\, P[X=y|x]\right]}_{Q(x,y)} p(x)\, dx$

$= \int_{x;f(x)=1} Q(x,y)\, p(x)\, dx + \int_{x;f(x)=0} Q(x,y)\, p(x)\, dx$ $\qquad L(0,0)=0$, $L(1,1)=0$

$= \int_{x;f(x)=1} (\overset{y=1}{Q_0(x) + 0})\, p(x)\, dx + \int_{x;f(x)=0} (\overset{y=0}{0 + Q_1(x)})\, p(x)\, dx$

b) $\mathrm{EPE}[f] = \int_x \left[I\{f(x)=0\} Q_1(x) + I\{f(x)=1\} Q_0(x)\right] p(x)\, dx$

$\mathrm{EPE}(f) - \int I\{f(x)=0\} [Q_1(x) - Q_0(x)]\, p(x)\, dx$

$= \int [I\{f(x)=1\} Q_0(x) + I\{f(x)=0\} Q_0(x)]\, p(x)\, dx)$

$= \int Q_0(x)\, p(x)\, dx = \int c_0 \underbrace{\Pr(Y=0 | X=x)\, p(x)}_{P(y,x)}\, dx$

$= c_0 \Pr(Y=y) = \mathrm{const}$ $\square$

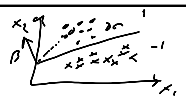
Instead of min EPE(f), we minimize $\int I\{f(x)=0\} [Q_1(x) - Q_0(x)]\, p(x)\, dx$

$\Rightarrow$ If $Q_1(x) - Q_0(x) > 0$: $f(x) = 1$

$Q_1(x) - Q_0(x) < 0$: $f(x) = 0$

$Q_1(x) > Q_0 \Rightarrow P(Y=1 | x) > \frac{c_0}{c_1} P(Y=0 | x)$

11)



$y_i \beta^T x_i > 0$ $\qquad \varepsilon = \min (y_i \beta^T z_i)$

$\Rightarrow y_i \beta^T z_i > 0$

$y_i \beta^T z_i \geq \varepsilon \Rightarrow y_i \underbrace{\left(\frac{1}{\varepsilon} \beta^T\right)}_{\beta_{sep}} z_i \geq 1 \Rightarrow y_i \beta_{sep}^T z_i \geq 1$